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APPLICATION OF THE CHARACTERISTIC METHOD IN CALCULATING THE TIME DEPENDENT, ONE-DIMENSIONAL, COMPRESSIBLE FLOW IN A TUBE WIND TUNNEL

Aero-Astrodynamics Laboratory

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

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John D. Warmbrod and Heinz G. Struck

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ABSTRACT

This report presents in detail a calculation method for one-dimensional, time-dependent flow through a pressure-tube wind tunnel. The computational procedure involves the method of characteristics for the one-dimensional unsteady flow of a perfect gas and accounts for area changes, shock formations, and intersection of discontinuities in the flow field.

A computer program written in Fortran language was constructed for the CDC 3200 digital computer and is presented along with a detailed description of the preparation of input for the program.

Calculated results of the program are presented for a pressure-tube wind tunnel now under construction at MSFC.

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AERO-ASTRODYNAMICS LABORATORY
RESEARCH AND DEVELOPMENT OPERATIONS

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DEFINITION OF SYMBOLS

Symbol	Definition
a	nondimensional speed of sound a^*/a_o^*
A	cross-sectional area
c_p	specific heat at constant pressure
f	nondimensional quantity $\frac{f^* L_o}{a_o^{*2}}$ where f^* is the sum of body and dissipative forces per unit mass
L_o	total length of facility (with dimensions)
M_S	Mach number of shock
M_{TEST}	Mach number in test section during steady state duration
p	nondimensional static pressure p^*/p_o^*
p_c	initial pressure ratio across diaphragm
P	right running characteristic variable
Q	left running characteristic variable
R	gas constant
S	nondimensional specific entropy $s/c_p(\gamma-1)$
t	nondimensional time $a_o^* t^*/L_o$
T	nondimensional temperature T^*/T_o^*
u	nondimensional flow velocity u^*/a_o^*
w_S	nondimensional shock velocity w_S^*/a_o^*
γ	ratio of specific heats
ρ	nondimensional density
ψ	nondimensional quantity $\frac{L_o\ a_o^*\ \psi^*}{\gamma A\ p_o^*}$ where ψ^* represents the mass flow removed through the walls per unit length

DEFINITION OF SYMBOLS (Continued)

Subscripts	Definition
o	reference conditions which for the case at hand were the conditions on the right side of the diaphragm before rupture
S	shock conditions
C.S.	contact surface conditions

Superscripts

*	dimensional quantities

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SUMMARY

This report presents in detail a calculation method for one-dimensional, time-dependent flow through a pressure-tube wind tunnel. The computational procedure involves the method of characteristics for the one-dimensional unsteady flow of a perfect gas and accounts for area changes, shock formations, and intersection of discontinuities in the flow field.

A computer program written in Fortran language was constructed for the CDC 3200 digital computer and is presented along with a detailed description of the preparation of input for the program.

Calculated results of the program are presented for a pressure-tube wind tunnel now under construction at MSFC.

I. INTRODUCTION

Ludwieg [7] first proposed the principle of the pressure tube wind tunnel and supervised the construction of the first facility of this type at the Aerodynamische Versuchsanstalt in Göttingen. Some experiments were reported in reference 8 of the stagnation pressure loss, and some schlieren pictures were made of the supersonic jet at the outlet of the nozzle. Calculations and measurements were given that show the extent of stagnation pressure losses in the test section with increasing boundary layer thickness. It was concluded that for large tube diameters (and therefore large Reynolds numbers) the limit length (stagnation pressure loss less than 1 percent due to the boundary layer) of the tube is approximately 100 tube diameters.

A supersonic pressure tube wind tunnel was constructed at the Royal Armament Research and Development Establishment in England during 1957, and some measurements from this wind tunnel are reported in reference 4. Reference 3 reported further measurements of the pressure at the nozzle end of the tube and static and pitot pressures in the working section of the nozzle. The diaphragm was located at the nozzle exit. Pictures of

the flow around a model were made by a high speed camera. An approximate calculation procedure for one-dimensional flow reported in reference 3 was used to obtain analytical results for running times and static pressures in the test section. Their analytical procedure assumed that the expansion fan emanated from the nozzle throat rather than from the diaphragm location, and thus neglected the unsteady expansion from the diaphragm through the nozzle throat. The effects of cross-sectional area changes were accounted for by steady state assumptions. An approximate method for calculating the boundary layer growth along the tube is also presented.

Reference 6, which is a good discussion on the solution of hyperbolic differential equations that describe one-dimensional, non-steady, compressible, multi-isentropic flow, presented three methods for solving the differential equations, along with the advantages and disadvantages associated with each method. Some hand calculations for an example of isentropic flow were made, but no extensive application of the procedures was performed.

Bull [1, 2] gives some measurements and calculations of the starting processes on an intermittent supersonic wind tunnel that were made at the Institute of Aerophysics at the University of Toronto. This tunnel consisted of a vacuum reservoir at the end of a Laval nozzle with a cellophane diaphragm located either upstream or downstream of the throat. Some calculations were made for one case assuming the flow to be time-dependent and one-dimensional, and a wave diagram was presented for the early phase of the flow from these calculations.

Rudinger [9] presents an excellent text on calculation procedures for solving the partial differential equations of non-steady, one-dimensional flow of compressible fluids through a duct. Because of the consistent and straightforward presentation of the solution techniques described in this book, many of the calculation procedures described in the present report were borrowed from this source.

Dahm [5] discussed a proposed Ludwieg-tube type of facility at the Marshall Space Flight Center for aerodynamic testing, at or near full scale Reynolds number, of a Saturn V rocket. This proposal generated an interest at the Center in developing an analytical capability for calculating start times and other properties of the flow pertinent to this facility, thus leading to the material presented here. Some experimental results of the starting characteristics for a small-scale pilot model of a blowdown wind tunnel that was tested at MSFC are presented in reference 11.

This paper presents a numerical procedure for solving the partial differential equations that describe the flow in a pressure-tube wind tunnel by a method generally referred to as the "method of characteristics." The mathematical model of the flow is assumed to be time-dependent, one-dimensional, multi-isentropic, and compressible. A computer program in Fortran IV language was formulated, and wave diagrams and other results of the flow that were calculated from this program are given.

The tunnel considered here (see Figure 1) consists of a long tube that serves as the storage reservoir, a convergent-divergent nozzle and test section, and an outlet into either the atmosphere or an emptying reservoir. The tube, closed at one end, is divided downstream of the test section by a diaphragm. When the diaphragm is ruptured, the high pressure gas on the left side of the diaphragm expands and compresses the gas on the low pressure side, thus creating a shock wave, contact surface, and an expansion fan (see Section IV). The contact surface is defined as the gas particles that were initially in contact with the diaphragm surface. The expansion fan, bounded on the left by what is termed here as the head wave and on the right by the tail wave, is mathematically the family of left-running characteristics. The solution consists of tracing the left- and right-running characteristic curves in the (x,t)-plane after the flow starts at time = 0.

The starting time is defined as the time when the flow properties in the test section become constant. The end test time is defined as the time that it takes the headwave to travel along the tube, reflect from the closed end of the tube, and reach the test section. Thus, the useful run time is the difference between the end test time and the start time. It is fairly evident that run times can be increased by increasing the length of the tube. Ludwieg [8] pointed out that to keep boundary layer effects negligible the tube length should not exceed approximately 100 tube diameters. Boundary layer effects can be approximately accounted for in the method presented in this report.

II. THE DIFFERENTIAL EQUATIONS

The main purpose of this report is to present the practical numerical procedures for solving the time-dependent equations for one-dimensional flow through a wind tunnel. Therefore, the fundamental differential equations which are derived in numerous references will be listed here only in the final form (see reference 9):

Continuity Equation

$$\frac{\partial(\rho^* A)}{\partial t^*} + \frac{\partial(\rho^* u^* A)}{\partial x^*} + \psi^* = 0. \tag{1}$$

Momentum Equation

$$\frac{Du^*}{Dt^*} = \frac{\partial u^*}{\partial t^*} + u^* \frac{\partial u^*}{\partial x^*} = - \frac{1}{\rho^*} \frac{\partial p^*}{\partial x^*} + f^*. \tag{2}$$

Equation of State

$$a^{*2} = \gamma \frac{p^*}{\rho^*} = \gamma R T^*. \tag{3}$$

Integrated Form of the First Law of Thermodynamics

$$s^* - s_o^* = c_p \ln \frac{T^*}{T_o^*} - c_p \frac{\gamma - 1}{\gamma} \ln \frac{p^*}{p_o^*} . \tag{4}$$

The subscript o indicates some state from which entropy changes are measured. The star superscript refers to dimensional quantities. For a definition of the symbols, the reader should refer to the Definition of Symbols. The underlying assumptions in the derivation of the above equations are:

(1) All quantities depend on the time t^* and a single coordinate x^*.

(2) There is only one velocity component u^* and that is in the x^*-direction.

(3) The gas follows the ideal gas laws, and the values of the specific heat are constant.

(4) All body and dissipative forces are lumped into a resultant force per unit mass, which is denoted in the momentum equation by f^*.

(5) Gas is permitted to leave the duct through the walls and is denoted in the continuity equation by ψ^*, which is defined as the mass flow through the walls per unit length.

It is customary to solve the above set of equations by a numerical integration procedure along a set of curves C in the independent variable (x,t)-plane. These curves are usually defined as particle paths and characteristic curves. Appendix A presents the derivation of the characteristic equations which are presented here in their final forms.

$$\frac{\delta_+ P}{\delta t} = -au\frac{\partial \ln A}{\partial x} - a\frac{\partial \ln A}{\partial t} + a\frac{\delta_+ S}{\delta t} + (\gamma - 1)\, a\frac{DS}{Dt} + f$$

$$- \psi a^{(\gamma-3)/(\gamma-1)}\, e^{\gamma(S-S_o)}. \tag{5}$$

$$\frac{\delta_- Q}{\partial t} = -au\frac{\partial \ln A}{\partial x} - a\frac{\partial \ln A}{\partial t} + a\frac{\delta_- S}{\delta t} + (\gamma - 1)\, a\frac{DS}{Dt} - f$$

$$- \psi a^{(\gamma-3)/(\gamma-1)}\, e^{\gamma(S-S_o)}. \tag{6}$$

The entropy condition which must be prescribed for any problem is given in a general form as

$$\frac{DS}{Dt} = F(a,u,S,x,t) \tag{7}$$

and this completes the system of equations for the three dependent variables a, u, and S. The prescribed entropy condition for the calculation procedures described in the subsequent sections of this report is given by

$$\frac{DS}{Dt} = 0 \tag{7a}$$

which characterizes the flow as multi-isentropic. This satisfies the condition that the entropy of each gas particle remains constant; however, different particles may have different entropies.

The special symbols for the differential operators are defined as

$$\delta_+ = \frac{\partial}{\partial t} + (u + a)\frac{\partial}{\partial x} \tag{8}$$

$$\delta_- = \frac{\partial}{\partial t} + (u - a) \frac{\partial}{\partial x} \tag{9}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} . \tag{10}$$

All quantities are nondimensional in the above equations (see Definition of Symbols). Equations (5) through (7) form a system of three linear first-order equations for three dependent variables, a, u, and S, that will be solved by means of a step-by-step procedure. The parameters P, Q, and S vary along curves in the (x,t)-plane that satisfy

$$\frac{dx}{dt} = u + a \qquad \text{for } P, \tag{11}$$

$$\frac{dx}{dt} = u - a \qquad \text{for } Q, \tag{12}$$

$$\frac{dx}{dt} = u \qquad \text{for } S. \tag{13}$$

The characteristic variables are defined by the relations

$$P = \frac{2}{\gamma - 1} a + u \tag{14}$$

and

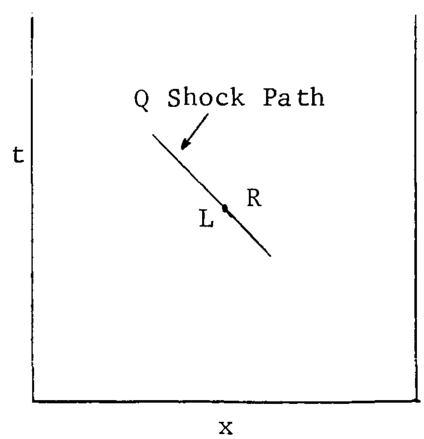
$$Q = \frac{2}{\gamma - 1} a - u. \tag{15}$$

The details of the calculation procedure for solving numerically the system of equations (5) through (7) are given in later sections of this report. The effects of boundary layer and mass flow through the walls of the duct were not considered, and the terms that represent those effects have been omitted from the equations.

III. THE NORMAL SHOCK RELATIONS

The characteristic equations which were derived from the basic differential equations in Appendix A are solved by taking small step-by-step increments in time, and the solution for each characteristic proceeds along its respective characteristic. These three characteristic curves were referred to as P in direction u + a, Q in direction u - a, and S in direction u. Since the characteristic S follows a curve of direction u, which is the particle path, two curves of this family can never cross. Whenever two curves of the same family (either P or Q) meet, a discontinuity in the pressure exists at this point. A boundary in the flow is thus established, and this boundary is defined as a normal shock wave. Two types of shock waves can therefore occur, either a P shock (converging of the P characteristics) or a Q shock (converging of the Q characteristics). The shock wave path will divide the wave diagram into two parts, and on each side certain conditions must be matched. Since it is assumed that changes of the flow variables across a shock wave take place instantaneously, the steady state relationships between flow variables on each side of the shock can be employed. The equations which relate the flow variables upstream and downstream of a stationary normal shock are generally referred to as the Rankine-Hugoniot equations. The derivation of these equations can be found in many references (e.g., [12, 13]), and therefore will not be repeated here. Since we are dealing with shocks that move with respect to the coordinate system, some modifications to the stationary shock relations are necessary. The equations appearing in this section are presented in their final form as they were used in the computational scheme. The shock Mach number M_S is defined here as the Mach number of a supersonic flow in which the shock would be stationary.

The relations across a Q shock point are as follows:



(1) Velocity

$$\frac{u_R - u_L}{a_L} = \frac{2(1-M_S^2)}{(\gamma+1)M_S} . \tag{16}$$

(2) Speed of Sound

$$\frac{a_R}{a_L} = \frac{\sqrt{[2+(\gamma-1)M_S^2][2\gamma M_S^2-(\gamma-1)]}}{(\gamma+1)M_S} . \tag{17}$$

(3) Q Characteristic

$$\frac{Q_R - Q_L}{a_L} = \frac{2}{\gamma - 1}\left(\frac{a_R}{a_L} - 1\right) - \frac{u_R - u_L}{a_L} . \tag{18}$$

(4) P Characteristic

$$\frac{P_R - P_L}{a_L} = \frac{2}{\gamma - 1}\left(\frac{a_R}{a_L} - 1\right) + \frac{u_R - u_L}{a_L} . \tag{19}$$

(5) Entropy

$$S_R - S_L = \frac{1}{\gamma(\gamma - 1)} \ln \left\{\left[\frac{2\gamma}{\gamma+1} M_S^2 - \frac{\gamma-1}{\gamma+1}\right]\left[\frac{1 + \frac{\gamma-1}{2} M_S^2}{\frac{\gamma+1}{2} M_S^2}\right]^{\gamma}\right\} . \tag{20}$$

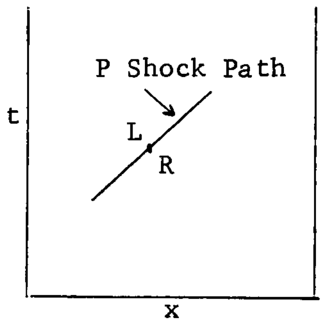
(6) Pressure

$$\frac{P_R}{P_L} = \frac{2\gamma}{\gamma + 1} M_S^2 - \frac{\gamma - 1}{\gamma + 1} . \tag{21}$$

(7) Shock Velocity Relative to the Duct

$$w_S = u_L - a_L M_S . \tag{22}$$

The subscripts L and R refer to the flow properties on the left- and right-hand sides of the shock point, and the subscript S refers to the shock. Analogous relations for a P shock point are:



(1) Velocity

$$\frac{u_R - u_L}{a_R} = \frac{2(1 - M_S^2)}{(\gamma + 1)M_S} . \tag{23}$$

(2) Speed of Sound

$$\frac{a_L}{a_R} = \frac{\sqrt{[2+(\gamma-1)M_S^2][2\gamma M_S^2-(\gamma-1)]}}{(\gamma+1)M_S} . \tag{24}$$

(3) Q Characteristic

$$\frac{Q_L - Q_R}{a_R} = \frac{2}{\gamma - 1}\left(\frac{a_L}{a_R} - 1\right) + \frac{u_R - u_L}{a_R} . \tag{25}$$

(4) P Characteristic

$$\frac{P_L - P_R}{a_R} = \frac{2}{\gamma - 1}\left(\frac{a_L}{a_R} - 1\right) - \frac{u_R - u_L}{a_R} . \tag{26}$$

(5) Entropy

$$S_L - S_R = \frac{1}{\gamma(\gamma-1)} \ln \left\{\left[\frac{2\gamma}{\gamma+1} M_S^2 - \frac{\gamma-1}{\gamma+1}\right]\left[\frac{1 + \frac{\gamma-1}{2} M_S^2}{\frac{\gamma+1}{2} M_S^2}\right]^{\gamma}\right\} . \tag{27}$$

(6) Pressure

$$\frac{p_L}{p_R} = \frac{2\gamma}{\gamma + 1} M_S^2 - \frac{\gamma - 1}{\gamma + 1} . \tag{28}$$

(7) Shock Velocity Relative to the Duct

$$w_S = u_R + a_R M_S . \tag{29}$$

The calculation procedure at a shock point involves the matching of the characteristic solution with the Rankine-Hugoniot solution at the shock point. From equations (18) and (26), it can be seen that the relations

$$\left(\frac{Q_R - Q_L}{a_L}\right)_{Q\ \text{shock}} = \left(\frac{P_L - P_R}{a_R}\right)_{P\ \text{shock}} = \Delta_S \tag{30}$$

where for the Rankine-Hugoniot solution,

$$\Delta_{S_{R.H.}} = \frac{2}{\gamma-1}\left\{\frac{\sqrt{[2+(\gamma-1)M_S^2][2\gamma M_S^2 - (\gamma-1)]}}{(\gamma+1)M_S} - 1\right\} - \frac{2(1-M_S^2)}{(\gamma+1)M_S}, \tag{31}$$

and, from the characteristic solution, $\Delta_{S_{CH.}}$ is calculated from equation (30) for the appropriate shock. The subscripts R.H. and CH. refer to the Rankine-Hugoniot and characteristic solutions, respectively. The condition that $\Delta_{S_{R.H.}} = \Delta_{S_{CH.}}$ at a shock point is satisfied by expressing equation (31) as

$$f(M_S) = \Delta_{S_{CH.}} - \frac{2}{\gamma-1}\left\{\frac{\sqrt{[2+(\gamma-1)M_S^2][2\gamma M_S^2 - (\gamma-1)]}}{(\gamma+1)M_S} - 1\right\} + \frac{2(1-M_S^2)}{(\gamma+1)M_S} = 0, \tag{32}$$

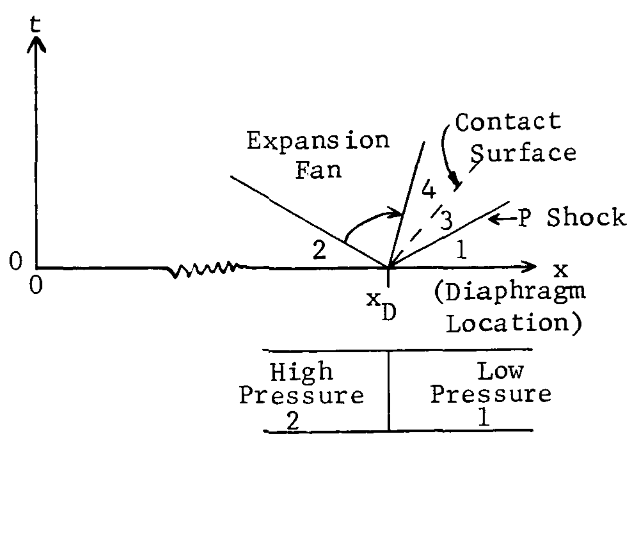
which, with $\Delta_{S_{CH.}}$ and γ given, is an algebraic equation for M_S. This equation is solved in the computer program by the Newton-Raphson method for finding the real roots of algebraic and transcendental equations. The iteration procedure to satisfy matching the solution of the characteristic equations with the Rankine-Hugoniot equations at a shock point is described in Section VII.

Some remarks should be made here about how the presence of a shock wave (P or Q) affects the flow and possibly the order of calculation as pertaining to the wave diagram. Assuming the gas to be flowing from left to right, the velocity of a P shock is supersonic relative to the velocity of the gas flow to the right; therefore, the P shock overtakes the flow of gas ahead of it. The flow conditions which lie to the right of the P shock path in the wave diagram are not then influenced by either the presence or strength of the P shock wave. The speed of a gas which is flowing from left to right will then be increased after passage of the P shock wave.

For a Q shock, the velocity of the flow is greater than the velocity of the shock relative to the duct, and therefore the flow conditions which lie to the left of the Q shock path in the wave diagram are independent of the strength or presence of the shock wave. It is seen, then, that, in case of a gas flowing from left to right, a Q shock will slow down the flow as it passes through the shock.

IV. THE INITIAL CONDITIONS

Consider a duct which is closed on the left end and which opens into the atmosphere or into an emptying reservoir on the right end divided by a diaphragm into two chambers. Let the pressure of the gas on the left-hand side of the diaphragm be higher than on the right, and furthermore, it is not restricted that the gases on both sides of the diaphragm be the same or have the same temperature. When the diaphragm is suddenly removed, the gas which was initially on the left side of the diaphragm expands and compresses the gas which was initially on the right-hand side, forming a P shock. The P shock travels through the gas which was initially on the right-hand side. Also created at diaphragm rupture is a contact surface. This contact surface is defined as the interface between the paths of the gas particles which were initially in contact with each side of the diaphragm. The problem at the instant the diaphragm is ruptured therefore consists of the simultaneous solution of the characteristic equations, the Rankine-Hugoniot equations, and the boundary conditions for the contact surface. It is assumed that the diaphragm is instantly removed, so the discussion that follows applies for time equals zero at the diaphragm location x_D.



Consider the duct shown in the illustration which is divided into two chambers by a diaphragm. Let the subscript 2 refer to the high pressure gas on the left-hand side of the diaphragm and the subscript 1 refer to the gas on the right-hand side. The prescribed initial conditions for the gases in the two chambers are the following:

(1) the pressure of the gases on each side p_2^* and p_1^*,

(2) the ratio of specific heat of the gases γ_2 and γ_1 and their gas constants R_2 and R_1,

(3) the temperatures of the gases T_2^* and T_1^*, and

(4) the geometry of the duct in a form such that the term $(dA/dx)/A$ can be calculated at any station, x, the location of the diaphragm, x_D^*, and the total length, L_o.

It is convenient to choose the initial conditions of the gas on the right-hand side of the diaphragm as the reference state. One has then the following values for the initial properties of the flow on the right- and left-hand sides, respectively.

$$
\begin{aligned}
a_1 &= \frac{a_1^*}{a_o^*} = 1 & \qquad a_2 &= \frac{a_2^*}{a_1^*} = \sqrt{\frac{\gamma_2 R_2 T_2^*}{\gamma_1 R_1 T_1^*}} \\
u_1 &= \frac{u_1^*}{a_o^*} = 0 & u_2 &= \frac{u_2^*}{a_1^*} = 0 \\
P_1 &= \frac{2}{\gamma_1 - 1} a_1 + u_1 & P_2 &= \frac{2}{\gamma_2 - 1} a_2 + u_2 \\
Q_1 &= \frac{2}{\gamma_1 - 1} a_1 - u_1 & Q_2 &= \frac{2}{\gamma_2 - 1} a_2 - u_2 \\
p_1 &= \frac{p_1^*}{p_o^*} = 1 & p_2 &= \frac{p_2^*}{p_1^*} \\
S_1 &= S_o = 0 & S_2 &= -\frac{1}{\gamma_2} \ln \left[\frac{p_2}{a_2^{(2\gamma_2)/(\gamma_2 - 1)}} \right] .
\end{aligned}
\tag{33}
$$

All distances are nondimensionalized by the total length of the duct, L_o, and the time is nondimensionalized by

$$
t = \frac{a_o^* t^*}{L_o} . \tag{34}
$$

The wave phenomenon that develops at diaphragm rupture is shown in the illustration. The details of calculating the initial solutions are given in the following steps:

(1) Assume that the velocity u_3 (the velocity between the contact surface and the P shock) has been given as a result of the previous iteration cycle or for the first iteration has been guessed. Since the velocity, u_1, and speed of sound, a_1, are known, the P shock Mach number can be calculated by the quadratic solution of equation (23) from the Rankine-Hugoniot relations:

$$M_S = \frac{-\left(\frac{u_1 - u_3}{a_1}\right)(\gamma_1 + 1) + \sqrt{\left[\left(\frac{u_1 - u_3}{a_1}\right)(\gamma_1 + 1)\right]^2 + 16}}{4} . \tag{35}$$

(2) Calculate the flow properties on the left side of the P shock by the Rankine-Hugoniot relations:

$$a_3 = \frac{a_1}{(\gamma_1 + 1)M_S} \sqrt{[2 + (\gamma_1 - 1)M_S^2][2\gamma_1 M_S^2 - (\gamma_1 - 1)]} \tag{36}$$

$$S_3 = \frac{1}{\gamma_1(\gamma_1 - 1)} \ln \left\{\left[\frac{2\gamma_1}{\gamma_1 + 1} M_S^2 - \frac{\gamma_1 - 1}{\gamma_1 + 1}\right]\left[\frac{1 + \frac{\gamma_1 - 1}{2} M_S^2}{\frac{\gamma_1 + 1}{2} M_S^2}\right]^{\gamma_1}\right\} \tag{37}$$

$$\frac{p_3}{p_1} = \frac{2\gamma_1}{\gamma_1 + 1} M_S^2 - \frac{\gamma_1 - 1}{\gamma_1 + 1} . \tag{38}$$

(3) One of the boundary conditions across the contact surface is satisfied by setting $p_4 = p_3$, and since the expansion takes place isentropically, the speed of sound on the left side of the contact surface is given by

$$a_4 = a_2(p_4/p_2)^{(\gamma_2 - 1)/2\gamma_2}. \tag{39}$$

(4) The velocity u_4 on the left side of the contact surface is given by the requirement that the characteristic variable P remains constant from the head-wave to the left side of the contact surface:

$$P_4 = P_2. \tag{40}$$

From the definition of P, the velocity u_4 can be calculated by

$$u_4 = P_4 - \frac{2}{\gamma_2 - 1} a_4. \tag{41}$$

(5) Satisfy the second boundary condition across the contact surface by setting $u_3 = u_4$ which yields a revised value for u_3. Steps (1) through (5) are repeated until the difference between the calculated values in u_3 for subsequent iteration cycles is less than a prescribed tolerance.

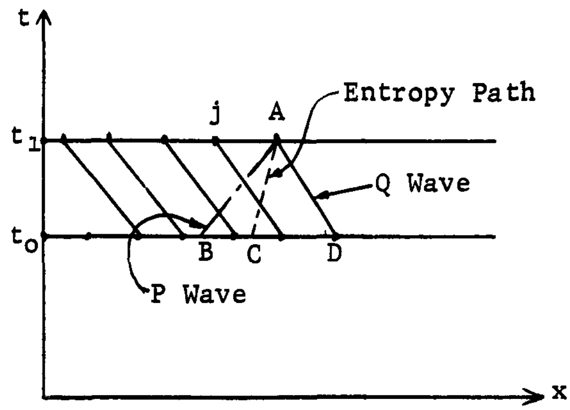
At the initial time $t = 0$, a centered expansion fan originates at the diaphragm location which is bounded on the left by the head-wave (Q_2 characteristic) and on the right by the tail-wave (Q_4 characteristic). Additional Q waves are introduced between the head- and tail-waves to keep the net of characteristics sufficiently fine; e.g., a total of n initial Q waves is employed in the computer program.

V. CHARACTERISTIC SOLUTION

In the numerical solution of the set of characteristic equations given in Section II, several procedures could be employed. The underlying process for solving these equations is, however, the same regardless of the particular procedure and is the integration of the characteristic equations along their respective paths in the x,t-plane. This integration is carried out by a step-by-step procedure along short straight line segments, the slopes of the respective characteristic variables being approximated by the average of their values at the end of the short line segments. The characteristic curves for the case at hand are designated as P, whose path in the x,t plane is in direction $u + a$, Q in direction $u - a$, and S in direction u. After exploring several different procedures, it was decided to use a constant time interval and to follow the path of one of the family of characteristics (Q), and to use an interpolation technique to follow the path of the other family (P) between subsequent time intervals. The order of calculation as performed by the computer program is from left to right along a line t = constant.

A. Calculation Procedure for a Regular Point

Assume that a distribution of n points has previously been computed or given as initial conditions on the line $t = t_o$ and that all properties of the flow are known at these points.



Let it also be assumed that the solution has been completed for j points on the line $t_1 = t_o + \Delta t$ (see illustration). Let the subscript A refer to properties at the j + 1 point on the line t_1 which is to be calculated, B refer to the point on the t_o line through which the P characteristic that intersects points A and B passes, C refer to the point on the t_o line through which the entropy path that intersects points A and C passes, and D is the origin of the Q characteristic from the t_o line which passes through point A. The calculation procedure consists of an iteration process for the location and properties at point A.

The straight line segment for the P wave that intersects points A and B in the time interval $t_1 - t_o = \Delta t$ must satisfy the relation

$$\frac{x_A - x_B}{\Delta t} = \frac{(u + a)_A + (u + a)_B}{2} , \tag{42}$$

which can be rewritten as

$$x_A - \frac{\Delta t}{2} (u + a)_A = x_B + \frac{\Delta t}{2} (u + a)_B . \tag{43}$$

The left side of the above equation relates the position and flow properties u_A and a_A at point A on line t_1 to the corresponding properties at point B on the line t_o. Keeping in mind that the flow properties are a known function of x on the line t_o as a result of the previous stage of calculation or as given initial conditions, then for given quantities of x_A, u_A, and a_A, we can find the point x_B and the flow properties at x_B by interpolation if a table of

$$z_1(x) = x + \frac{\Delta t}{2} (u + a) \tag{44}$$

has been prepared for the distribution of points on the t_o line.

Following the same approach for the entropy path used for the P wave, the straight line segment for the entropy path in the time interval must satisfy the relation

$$\frac{x_A - x_c}{\Delta t} = \frac{u_A + u_c}{2}, \tag{45}$$

which can be written as

$$x_A - \frac{\Delta t}{2} u_A = x_C + \frac{\Delta t}{2} u_C. \tag{46}$$

A table of values for

$$z_2(x) = x + \frac{\Delta t}{2} u \tag{47}$$

is also assumed to have been prepared and stored at the completion of the calculations for the t_0 line. Since the procedure follows each Q characteristic from one time interval to the next, an analogous table for the Q wave is not necessary.

The step-by-step procedure for calculating the flow properties and position of point A is as follows:

(1) Assuming that u_A and a_A have been given from a previous step, or for the first iteration have been guessed, the location of x_A is given by intersection of the line segment for the Q-wave path that passes through x_D with the constant time line, $t = t_1$.

$$x_A = x_D + \frac{\Delta t}{2} [(u - a)_A + (u - a)_D]. \tag{48}$$

(2) Calculate the left-hand sides of equations (43) and (46):

$$\begin{aligned} c_1 &= x_A - \frac{\Delta t}{2} (u + a)_A \\ c_2 &= x_A - \frac{\Delta t}{2} u_A, \end{aligned} \tag{49}$$

whereby the points x_B and x_C can be found by an inverse interpolation of the prepared tables of $z_1(x)$ and $z_2(x)$. The flow properties u, a, and S are then given for these two points by interpolation. Values for the characteristic variables P and Q are calculated by equations (14) and (15) for these two points.

(3) The characteristic variables P_A and Q_A are then calculated by

$$P_A = P_B + \left[\frac{(-au\frac{1}{A}\frac{dA}{dx})_B + (-au\frac{1}{A}\frac{dA}{dx})_A}{2}\right]\Delta t + \frac{a_A + a_B}{2}(S_A - S_B) \tag{50}$$

$$Q_A = Q_D + \left[\frac{(-au\frac{1}{A}\frac{dA}{dx})_D + (-au\frac{1}{A}\frac{dA}{dx})_A}{2}\right]\Delta t + \frac{a_A + a_D}{2}(S_A - S_D)$$

where the geometrical term $\frac{1}{A}\frac{dA}{dx}$ is assumed to be a known function of x.

(4) From the definition of the characteristic variables, equations (14) and (15), new values of the velocity u_A and speed of sound a_A can be calculated.

$$u_A = \frac{1}{2}(P_A - Q_A) \tag{52}$$

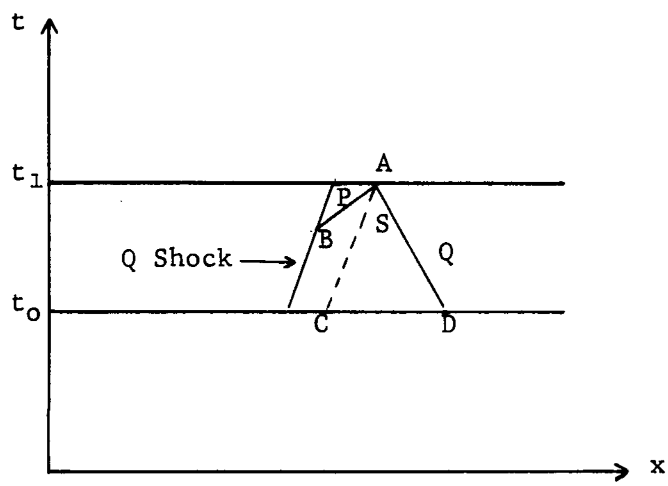
$$a_A = \frac{\gamma - 1}{4}(P_A + Q_A). \tag{53}$$

(5) Steps (1) through (4) are repeated until the differences of u_A and a_A for subsequent iteration cycles are less than a prescribed tolerance.

After the convergence is met, the final values of the flow properties from the last iteration cycle are stored for use in calculating the solutions at the next time interval. All interpolations that were carried out in the above steps are linear.

B. Calculation Procedure for Points Near Boundaries

The procedure for calculating a regular point given under Section V(A) involved projecting a Q wave from time t_o to time t_1 along its respective characteristic, and by an interpolation process finding the point at the previous time interval where the P characteristic passes that intersects with the Q wave at the time t_1. It is obvious that this standard procedure cannot be applied for calculating points very near to the right side of a boundary or discontinuity since the Q wave and entropy path can cross the boundary in the time interval. For example, the illustration shows the intersection of the P and Q characteristics along with the entropy path at point A. Since the P characteristic intersects the Q shock at point B, the standard procedure given in Section V(A) is not applicable.



An alternate procedure was used for calculating points on the right side of a boundary where the P characteristic or entropy path crosses the boundary in the time interval under consideration. Consider the path of the boundary between times t_o and t_1 along with the flow properties on the right-hand side of the shock at times t_o and t_1 to be known. The equations that must be solved to determine the location and flow properties at point A are:

$$\frac{x_A - x_B}{t_1 - t_B} = \frac{(u + a)_A + (u + a)_B}{2} \quad \text{along the P wave,} \qquad (54)$$

$$\frac{x_A - x_D}{\Delta t} = \frac{(u - a)_A + (u - a)_D}{2} \quad \text{along the Q wave,} \tag{55}$$

$$\frac{x_A - x_C}{t_1 - t_C} = \frac{u_A + u_C}{2} \quad \text{along the entropy path,} \tag{56}$$

$$S_A = S_C, \tag{57}$$

$$P_A = P_B + \left[\frac{\left(-au \frac{1}{A} \frac{dA}{dx}\right)_A + \left(-au \frac{1}{A} \frac{dA}{dx}\right)_B}{2}\right] (t_1 - t_B)$$

$$+ \frac{a_A + a_B}{2} (S_A - S_B), \tag{58}$$

$$Q_A = Q_D = \left[\frac{\left(-au \frac{1}{A} \frac{dA}{dx}\right)_A + \left(-au \frac{1}{A} \frac{dA}{dx}\right)_D}{2}\right] \Delta t + \frac{a_A + a_D}{2} (S_A - S_D). \tag{59}$$

The equation for any point B on the path of the boundary between times t_0 and t_1 is given by

$$x_{R,1} - x_B = \left(\frac{x_{R,1} - x_{R,0}}{\Delta t}\right) (t_1 - t_B), \tag{60}$$

where the subscripts R,0 and R,1 refer to properties on the right-hand side of the boundary at times t_0 and t_1. Combining equations (54) and (60) results in

$$t_1 - t_B = \frac{x_A - x_{R,1}}{\frac{(u + a)_A + (u + a)_B}{2} - \frac{x_{R,1} - x_{R,0}}{\Delta t}}, \tag{61}$$

which relates the intersection of the P wave with the boundary. Replacing the subscript B in equation (60) by the subscript C and combining this equation with equation (56) results in

$$t_1 - t_C = \frac{x_A - x_{R,1}}{\frac{u_A + u_C}{2} - \frac{x_{R,1} - x_{R,0}}{\Delta t}}, \tag{62}$$

which relates the intersection of the entropy path with the boundary.

The step-by-step procedure for solving the above set of equations for the position and flow properties at point A is as follows:

(1) Assume that u_A and a_A are known from the previous iteration step or for the first iteration have been guessed. Equation (55) gives the relation for x_A

$$x_A = x_D + \frac{\Delta t}{2} [(u - a)_A + (u - a)_D]. \tag{63}$$

(2) The point of intersection of the P wave and the boundary is found by solving equation (61) by the iteration process, i.e.,

(a) guess t_B,

(b) obtain u_B and a_B by a linear interpolation of these known properties on the right side of the boundaries at times t_o and t_1, and

(c) calculate an improved value of t_B from equation (61).

Steps (b) and (c) are repeated until changes in t_B satisfy a prescribed tolerance. The axial location x_B is then calculated by equation (54), and S_B is given by interpolation as was done for u_B and a_B in step (b).

(3) The point of intersection of the entropy path and the boundary is given by solving equation (62) by the same method as described above for equation (61). The entropy S_C is then given by interpolation of the known values of S at $x_{1,R}$ and $x_{0,R}$. The entropy at point A is then given by (57). If $t_C \leqq t_o$, the entropy is handled in the same way as described in Section V, paragraph A, for a regular point.

(4) Values for the characteristic variables at point A are given by equations (58) and (59) from which improved values of u_A and a_A can be calculated.

$$a_A = \frac{\gamma - 1}{4} (P_A + Q_A) \tag{64}$$

$$u_A = \frac{1}{2} (P_A - Q_A). \tag{65}$$

Steps (1) through (4) are repeated until u_A and a_A converge to a final value within a prescribed tolerance.

VI. CALCULATION PROCEDURE FOR A CONTACT SURFACE

A contact surface is defined as the interface between the paths of two different gases, or the same gas at different entropy levels, in which no flux of matter passes. In the x,t-plane the contact surface is a line of discontinuity in which the speed of sound and the entropy are discontinuous. By nature of its definition, the flow velocity and pressure through the contact surface at a point in the x,t-plane are constant.

Before entering into a discussion of the iteration procedure for calculating the flow properties on each side of the contact surface, some relations which will be required are derived. Let the subscripts L and R refer to flow conditions on the left and right sides of the contact surface, respectively. The characteristics P on the left and Q on the right are, by definition,

$$P_L = \frac{2}{\gamma_L - 1} a_L + u_L \tag{66}$$

$$Q_R = \frac{2}{\gamma_R - 1} a_R - u_R. \tag{67}$$

By applying the boundary condition $u_L = u_R$, equations (66) and (67) combine to give

$$P_L + Q_R = \frac{2}{\gamma_L - 1} a_L + \frac{2}{\gamma_R - 1} a_R. \tag{68}$$

The pressures on each side of the contact surface are expressed as

$$p_L = a_L^{2\gamma_L/(\gamma_L-1)} e^{-\gamma_L S_L} \tag{69}$$

$$p_R = a_R^{2\gamma_R/(\gamma_R-1)} e^{-\gamma_R S_R} \tag{70}$$

which includes the condition that the reference entropy level is zero. Applying the boundary condition $p_L = p_R$, equations (69) and (70) combine to give

$$a_R = \left\{a_L^{2\gamma_L/(\gamma_L-1)} e^{\gamma_R S_R - \gamma_L S_L}\right\}^{(\gamma_R-1)/2\gamma_R} . \tag{71}$$

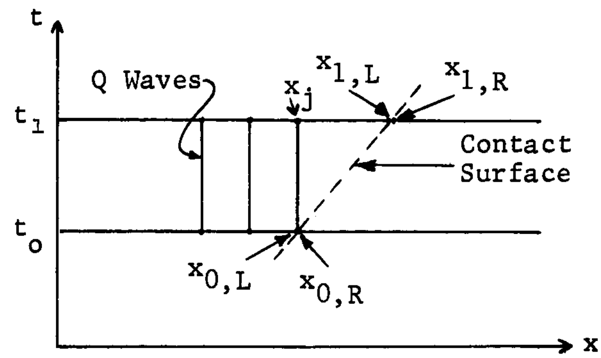
When equations (68) and (71) are combined, the satisfaction of both boundary conditions across the contact surface is included. We then obtain

$$P_L + Q_R = \frac{2}{\gamma_L - 1} a_L + \frac{2}{\gamma_R - 1} a_L^{\left[\gamma_L(\gamma_R-1)\right]/\left[\gamma_R(\gamma_L-1)\right]} e^{\frac{\gamma_R-1}{2\gamma_R}(\gamma_R S_R - \gamma_L S_L)} \tag{72}$$

which, for P_L, Q_R, S_R, S_L, γ_L, and γ_R given, is a transcendental equation. This equation must be solved numerically for a_L. For $\gamma_L = \gamma_R = \gamma$, it can be solved directly to give

$$a_L = \frac{\frac{\gamma - 1}{2}(P_L + Q_R)}{1 + e^{\frac{\gamma-1}{2}(S_R - S_L)}} . \tag{73}$$

It is assumed that the solution is given for a distribution of n points on the t_o line and that a contact surface passes through this line. Also assumed is that j regular points have been calculated on the t_1 line. The procedure for calculating the contact surface point on the t_1 line is given in the following steps.



(1) The values for the entropy on each side of the contact surface are given since the entropy level for each side remains constant throughout the trajectory of the contact surface in the x,t-plane; therefore, set

$$S_{1,L} = S_{0,L}$$

$$S_{1,R} = S_{0,R},$$

where the first subscript 0 or 1 refers to the t_o or t_1 line, respectively.

(2) Assuming that $u_{1,L}$ and $a_{1,L}$ have been given from the previous iteration cycle, or for the first iteration cycle, have been guessed, calculate the x location of the contact surface point on the t_1 line:

$$x_{1,L} = x_{1,R} = x_{0,L} + \frac{\Delta t}{2} (u_{1,L} + u_{0,L}). \tag{74}$$

(3) Satisfy the boundary condition of no velocity change across the contact surface at a point by setting $u_{1,R} = u_{1,L}$ and satisfying the analogous boundary condition for pressure by calculating the speed of sound on the right-hand side from equation (71).

(4) Find the points on the t_o line through which the P and Q waves that intersect at $x_{1,L} = x_{1,R}$ pass. This is accomplished by an interpolation process that locates the x stations on the t_o line that satisfy

$$\frac{x_{1,L} - x_P}{\Delta t} = \frac{(u + a)_{1,L} + (u + a)_P}{2} \tag{75}$$

for the P wave, and

$$\frac{x_{1,R} - x_Q}{\Delta t} = \frac{(u - a)_{1,R} + (u - a)_Q}{2} \tag{76}$$

for the Q wave. The subscripts P and Q refer to properties at the points x_P and x_Q on the t_o line which satisfy the above relations.

(5) Calculate values for $P_{1,L}$ and $Q_{1,R}$ by way of the characteristics equations

$$P_{1,L} = P_P + \left[\frac{(-au \frac{1}{A} \frac{dA}{dx})_P + (-au \frac{1}{A} \frac{dA}{dx})_{1,L}}{2}\right] \Delta t + \left(\frac{a_P + a_{1,L}}{2}\right) (S_{1,L} - S_P) \tag{77}$$

and

$$Q_{1,R} = Q_Q + \left[\frac{(-au \frac{1}{A} \frac{dA}{dx})_Q + (-au \frac{1}{A} \frac{dA}{dx})_{1,R}}{2}\right] \Delta t + \left(\frac{a_Q + a_{1,R}}{2}\right) (S_{1,R} - S_Q). \tag{78}$$

(6) Calculate an improved value for $a_{1,L}$ by solving equation (72), or by solving (73) for $\gamma_L = \gamma_R$, and an improved value for $u_{1,L}$ by

$$u_{1,L} = P_{1,L} - \frac{2}{\gamma_L - 1} a_{1,L}. \tag{79}$$

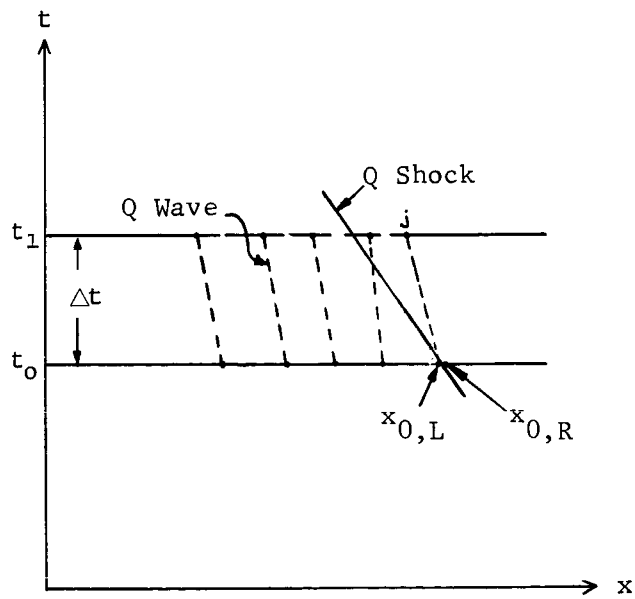
Steps (2) through (6) are repeated until both $u_{1,L}$ and $a_{1,L}$ converge to constant values within a prescribed tolerance.

VII. CALCULATION PROCEDURE FOR A SHOCK POINT

As suggested by Hartree [6], it was found that the treatment of boundaries and discontinuities was simpler when using a constant time interval rather than following a grid of characteristics. This method, explained in detail in Section V for a general point, follows one of the family of characteristics (Q waves) from time t_0 to time $t_1 = t_0 + \Delta t$ and interpolates the x station at time t_0 through which the other characteristics (P waves) pass. The order of the calculations proceeds from left to right in the wave diagram for each line t = constant. Because of the order and method by which the calculations are carried out, the detailed procedure used to calculate a Q shock point is somewhat different from that for a P shock point. The details of the procedure for each type of shock point will therefore be presented separately.

A. Q Shock Point

It is assumed that the solution has been completed for a distribution of n points on the line $t = t_0$ and that a Q shock passes through this line. Since changes of the flow variables across the shock are assumed to take place instantaneously and since the shock thickness is assumed to be zero, the properties of the flow at the shock point are double-valued. To explain the procedure for calculating a Q shock point, a double subscript of notation is used. Let the first subscripts 0 and 1 refer to the shock at times t_0 and $t_0 + \Delta t$ and the second subscripts L and R refer to the left- and right-hand sides of the shock, respectively. It is also assumed that j regular points which follow Q characteristics have been calculated on the $t_0 + \Delta t$ line where the j-th point is the point generated from the point on the immediate left side of the shock wave on the t_0 line. If we keep in mind that Q characteristics overtake a Q shock from both sides, the illustration shows Q waves that have crossed the shock path from the left side. After the iteration for the shock point has been completed, those points that lie on the right-hand side of the shock at time t_1 are discarded. These points were retained only for interpolation of properties on the left side of the shock during the iteration procedure for the shock point.



The calculation procedure for the Q shock point on the t_1 line is now given in the order that the computer program actually solves the problem:

(1) Guess the shock velocity $w_{1,S}$ at time $t_1 = t_o + \Delta t$. Calculate the average shock velocity between times t_o and t_1:

$$\bar{w} = \frac{w_{0,S} + w_{1,S}}{2} .$$

(2) Calculate the position of the shock point on the t_1 line:

$$x_{1,L} = x_{1,R} = x_{0,L} + \Delta t \cdot \bar{w}. \tag{80}$$

(3) Interpolate for the properties $u_{1,L}$, $a_{1,L}$ and $S_{1,L}$ on the left side of the shock point.

(4) Calculate the shock Mach number:

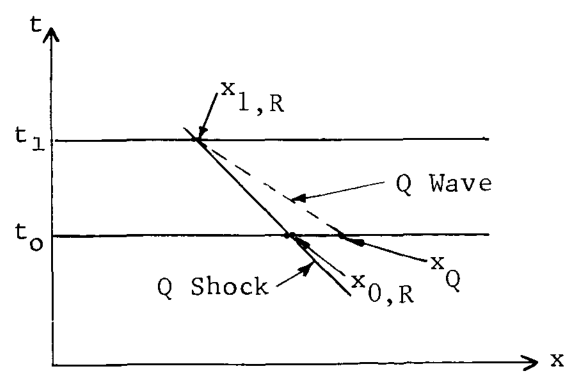
$$M_{1,S} = \frac{u_{1,L} - w_{1,S}}{a_{1,L}} . \tag{81}$$

(5) Calculate the properties on the right side of the Q shock point which are given by the Rankine-Hugoniot equations presented in Section III.

(6) The point x_Q on the t_o line through which the Q characteristic passes that also passes through the point $x_{1,R}$ is found by satisfying

$$\frac{x_{1,R} - x_Q}{\Delta t} = \frac{(u - a)_{1,R} + (u - a)_Q}{2} . \tag{82}$$

This equation is solved by the method of iteration for x_Q.



(7) Calculate a new value for the Q characteristic on the right side of the shock point by

$$Q_{1,R} = Q_Q + \left[\frac{(-au\,\frac{1}{A}\,\frac{dA}{dx})_Q + (-au\,\frac{1}{A}\,\frac{dA}{dx})_{1,R}}{2}\right]\Delta t + \frac{a_{1,R}+a_Q}{2}(S_{1,R}-S_Q). \quad (83)$$

(8) The following relationship across the shock is now calculated:

$$\Delta_S = \frac{Q_{1,R} - Q_{1,L}}{a_{1,L}}. \quad (84)$$

This relationship yields a new value for the shock Mach number $M_{1,S}$ through the Rankine-Hugoniot equation (32).

(9) A new shock velocity $w_{1,S}$ is then given by

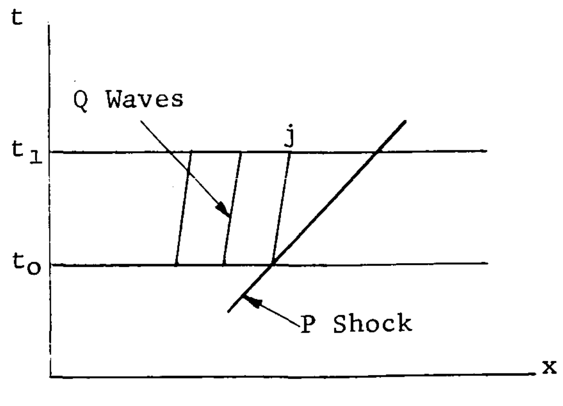
$$w_{1,S} = u_{1,L} - a_{1,L}\,M_{1,S}. \quad (85)$$

A comparison of the guessed shock velocity with the calculated shock velocity determines whether a prescribed tolerance in their difference has been met. Steps (1) through (9) are repeated by replacing the guessed value of $w_{1,S}$ by the calculated value until the tolerance is met.

All points, previously calculated on the line t_1, which lie on the right-hand side of the shock point are now dropped. Calculation of points on the right of a boundary is discussed in Section V, Paragraph B, of this report.

B. P Shock Point

The calculation procedure for a P shock point is somewhat different from that of a Q shock point, but the underlying process of matching the Rankine-Hugoniot solution with the characteristic solution at the shock point is the same. Again it is assumed that the calculation proceeds from left to right and that j points have been calculated on the t_1 line as shown in the illustration. The detailed procedure for calculating the P shock point is given in the following steps.



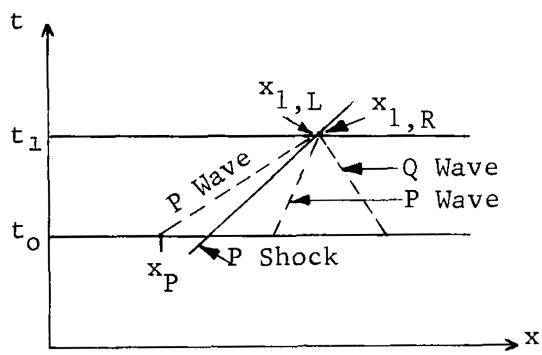
(1) Guess the shock velocity $w_{1,S}$ and calculate the average shock velocity between times t_o and t_1:

$$\bar{w} = \frac{w_{0,S} + w_{1,S}}{2} .$$

(2) Calculate the position of the shock point on the t_1 line:

$$x_{1,L} = x_{1,R} = x_{0,L} + \Delta t \cdot \bar{w}. \tag{86}$$

(3) Find the locations on the t_o line through which the P and Q characteristics pass that intersect at the point $x_{1,R}$. This is accomplished by approximately the same process as that for calculating a general point, and therefore a step-by-step explanation of the procedure will not be repeated here. The difference is that a particular Q characteristic is not followed; therefore, interpolations at time t_o are necessary to find both the P and Q characteristics that pass through the point $x_{1,R}$. This step yields the flow variables on the right-hand side of the P shock.



(4) By knowing the flow variables on the right side of the shock point from the above step, the P shock Mach number is calculated by

$$M_{1,S} = \frac{w_{1,S} - u_{1,R}}{a_{1,R}} . \tag{87}$$

(5) The Rankine-Hugoniot relations (Section III) give the flow variables on the left-hand side of the P shock.

(6) The point x_P on the t_o line through which the P characteristic passes (see illustration) and that also passes through the point $x_{1,L}$ is found by satisfying

$$\frac{x_{1,L} - x_P}{\Delta t} = \frac{(u + a)_P + (u + a)_{1,L}}{2} . \tag{88}$$

This equation is solved by the method of iteration for x_P.

(7) Calculate a new value for the P characteristic on the left-hand side of the P shock by

$$P_{1,L} = P_p + \left[\frac{(-au \frac{1}{A}\frac{dA}{dx})_P + (-au \frac{1}{A}\frac{dA}{dx})_{1,L}}{2}\right] \Delta t + \left(\frac{a_{1,L} + a_P}{2}\right)(S_{1,L} - S_p). \tag{89}$$

(8) Calculation of the relationship

$$\Delta_S = \frac{P_{1,L} - P_{1,R}}{a_{1,R}} \tag{90}$$

across the shock yields a new value for $M_{1,S}$ by way of the Rankine-Hugoniot equation (32).

(9) A new shock velocity $w_{1,S}$ is calculated by

$$w_{1,S} = u_{1,R} + a_{1,R} M_{1,S}. \tag{91}$$

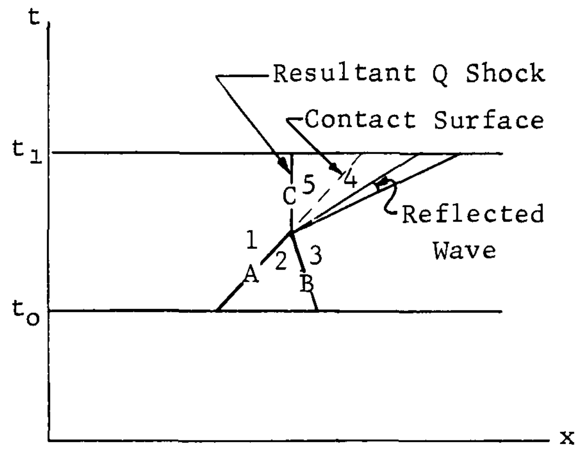
Comparison of the guessed value with the calculated value of the shock velocity determines whether a prescribed tolerance has been met for their difference. Steps (1) through (9) are repeated by replacing the guessed value with the calculated value until this tolerance is met.

VIII. INTERACTION OF DISCONTINUITIES

Thus far in this report, we have dealt only with cases where characteristics cross or intersect; however, it can easily be seen that there are many other possibilities of boundaries and discontinuities intersecting with one another, such as shocks of like families intersecting, shocks of unlike families intersecting, and shock contact surface intersections. Although the general methods for handling these interactions are similar, each case requires somewhat different calculation procedures. Because of the large number of possibilities that can occur, only the intersection of the discontinuities that were encountered in the problem at hand will be discussed. The reader should be able to easily modify the discussed procedure for a particular interaction not treated herein.

A. Merging of Two Q Shocks

When one Q shock overtakes another, the wave diagram in the vicinity of the point of intersection of the two Q shocks appears as shown in the illustration. A Q shock stronger than either of the merging shocks results. Also produced is a contact surface which is the particle path that passes through the intersection point and separates the gases that have been compressed by the two shocks. For $\gamma \leq 5/3$ (see reference 10), a reflected wave which was created by the interactions was found to be quite weak for all of the shock-shock interactions encountered in this problem.



The interaction of two Q shocks was solved by an approximate trial-and-error procedure. Assume that two Q shocks have been detected to cross between times t_o and t_1. Of course, there must be some logical check to detect this crossing in the computer program, but, as there are numerous ways this could be handled, this will not be discussed here. Let x_I, t_I refer to the point of intersection of the two Q shocks, the subscripts 1 through 5 refer to flow properties of the regions in the vicinity of the interaction as shown in the illustration, and A, B, and C refer to the three Q shocks. The iteration procedure is as follows:

(1) Consider that the velocity u_5 has been given from the previous iteration step or for the first iteration cycle has been guessed. It is assumed that the flow properties of Region 1 are given by the values of the flow which had been calculated on the left side of shock A at time t_o, and that the property values of region 3 are those which were calculated for the right-hand side of shock B at time t_o. These assumptions are justified because of the smallness of the time step Δt.

(2) Properties in region 5 can then be calculated from the Rankine-Hugoniot relations across a Q shock:

$$\Delta u_C = \frac{u_5 - u_1}{a_1} , \tag{92}$$

$$M_C = \frac{-\frac{\gamma + 1}{2}\Delta u_C + \sqrt{\left(\frac{\gamma + 1}{2}\Delta u_C\right)^2 + 4}}{2}, \tag{93}$$

$$a_5 = a_1 \frac{\sqrt{[2 + (\gamma - 1)M_C^2][2\gamma M_C^2 - (\gamma - 1)}}{(\gamma + 1)M_C}, \tag{94}$$

$$S_5 = S_1 + \frac{1}{\gamma(\gamma - 1)} \ln \left\{\left[\frac{2\gamma}{\gamma+1} M_C^2 - \frac{\gamma-1}{\gamma+1}\right]\left[\frac{1 + \frac{\gamma-1}{2} M_C^2}{\frac{\gamma+1}{2} M_C^2}\right]^{\gamma}\right\}. \tag{95}$$

(3) Since the reflected wave was a centered expansion fan for the range of γ possible for this problem, the entropy across the expansion is constant; i.e., $S_4 = S_3$. The speed of sound for region 4 is then given by satisfying the boundary condition of equal pressure across the contact surface:

$$a_4 = a_5 e^{\frac{\gamma-1}{2}(S_4-S_5)}. \tag{96}$$

(4) The reflected wave, which is always an expansion wave for $\gamma \leqq 5/3$, separates region 3 from region 4, and the value of the characteristic variable Q across the reflected wave is constant. Therefore, by setting $Q_4 = Q_3$, the velocity in region 4 is given by

$$u_4 = \frac{2}{\gamma - 1} a_4 - Q_4. \tag{97}$$

(5) An improved value for u_5 is given by satisfying the second boundary condition across the contact surface:

$$u_5 = u_4. \tag{98}$$

Steps (2) through (5) are repeated until the difference in calculated values of a_5 for subsequent iteration cycles is less than a prescribed tolerance.

The shock velocity of shock C is then given by

$$w_C = u_1 + a_1 M_C, \tag{99}$$

the location of the Q shock point on the line t_1 is

$$x_C(t_1) = x_I + w_C(t_1 - t_I), \tag{100}$$

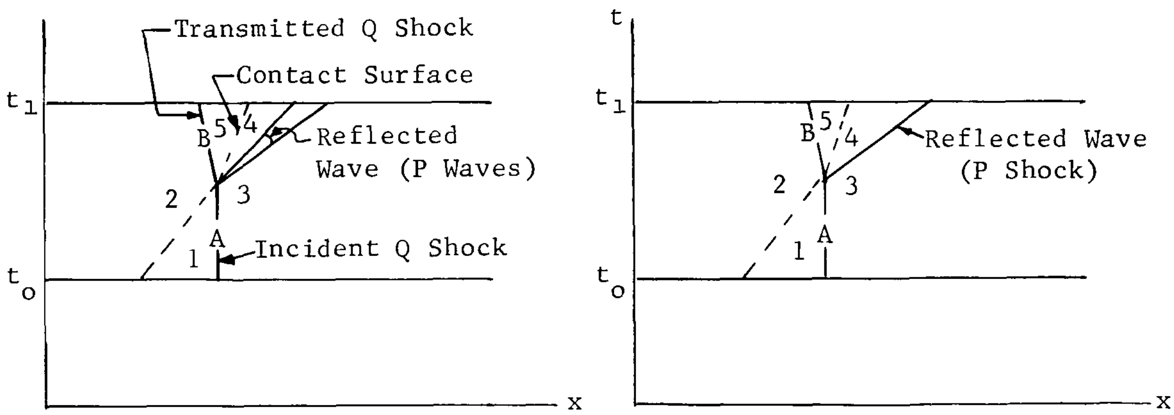
and the location of the contact surface point on the line t_1 is

$$x_{C.S.}(t_1) = x_I + u_5(t_1 - t_I). \tag{101}$$

The reflected wave was found to be very weak for all the interactions of two Q shocks that occurred in the problem. Points on the right-hand side of the contact surface on line t_1 were calculated in the same manner as the points near boundaries (see Section V, Paragraph B). This, in effect, spreads the centered expansion, which was weak over the time interval Δt rather than at a point. This approximate way of handling the reflected wave should be quite good since the time step is small.

B. Crossing of a Q Shock by a Contact Surface

The following illustrations show the two types of interactions that can occur when a contact surface intersects with a Q shock.



If the pressure ratio across the transmitted Q shock p_5/p_2 is less than the pressure ratio across the incident Q shock p_3/p_1, then the reflected wave is a rarefaction fan. However, if $p_5/p_2 > p_3/p_1$, the reflected wave is a P shock. In the cases calculated for the problem at hand in which a contact surface crossing a Q shock occurred, the reflected wave was always a weak rarefaction fan. The computer program was coded for the case of only the reflected wave being a rarefaction fan, but maintaining the capability of detecting if the reflected wave is a P shock and printing out a message describing this condition and the strength of the P shock. The rarefaction fan is handled in the same manner as that described in the previous section when two Q shocks interact.

The interaction of the contact surface and a Q shock was solved by an approximate trial-and-error procedure. Let x_I, t_I refer to the intersection point, the subscripts 1 through 5 refer to the regions indicated in the illustrations, and A and B refer to the incident and transmitted Q shocks, respectively. Flow properties in regions 1, 2, and 3 are assumed known. Calculation steps for solving the interaction process are as follows:

(1) Consider that the velocity in region 5 (u_5) has been given from the previous iteration cycle or for the first cycle has been guessed.

(2) The remaining properties in region 5 can then be calculated from the Rankine-Hugoniot relations across a Q shock:

$$\Delta u_B = \frac{u_5 - u_2}{a_2} \tag{102}$$

$$M_B = \frac{-\frac{\gamma+1}{2}\Delta u_B + \sqrt{\left(\frac{\gamma+1}{2}\Delta u_B\right)^2 + 4}}{2} \tag{103}$$

$$a_5 = a_2 \frac{\sqrt{[2 + (\gamma - 1)M_B^2][2\gamma M_B^2 - (\gamma - 1)]}}{(\gamma + 1)M_B} \tag{104}$$

$$S_5 = S_2 + \frac{1}{\gamma(\gamma - 1)} \ln \left\{\left[\frac{2\gamma}{\gamma+1} M_B^2 - \frac{\gamma-1}{\gamma+1}\right]\left[\frac{1 + \frac{\gamma-1}{2} M_B^2}{\frac{\gamma+1}{2} M_B^2}\right]^{\gamma}\right\} . \tag{105}$$

(3) Since it has been assumed that the reflected wave is a rarefaction fan, there is no change in entropy across the reflected wave; i.e., $S_4 = S_3$. The speed of sound is then given by satisfying the boundary condition of equal pressure through the contact surface.

$$a_4 = a_5 e^{\frac{\gamma-1}{2}(S_4-S_5)} . \tag{106}$$

(4) An additional condition that must be satisfied is the constancy of the characteristic variable Q across the expansion fan

$$Q_4 = Q_3$$

which by definition yields the relationship for the velocity in region 4.

$$u_4 = \frac{2}{\gamma - 1} a_4 - Q_4. \tag{107}$$

(5) The second boundary condition across the contact surface is now satisfied by setting

$$u_5 = u_4$$

which then provides the improved value for u_5 in the next iteration cycle.

Steps (2) through (5) are repeated until the prescribed tolerance for the difference in successive values of u_5 is met.

The location of the shock and contact surface points, and the calculation of points on the right side of the contact surface by the characteristic solution at time t_1, follow the same procedure described in the previous section when two Q shocks interact.

IX. EXIT CONDITIONS

After diaphragm rupture, the volume of gas that has been compressed on the left-hand side of the diaphragm flows through the facility and empties into a reservoir which is connected to the end of the duct.

Since the mathematical solution is intended to calculate only the early development of the flow, it is not necessary to include the conditions for inflow at the end of the duct. The possibility of inflow at the exit could exist only at a very late time because of a buildup of pressure in the emptying reservoir, or at a very late stage of the blowdown if emptying into the atmosphere.

For the problem at hand, gas is always assumed to be leaving the duct at the exit station L_o and is therefore treated as an outflow problem. The gas in the external region is assumed to be at rest. The calculation procedure for satisfying the appropriate boundary conditions for subsonic, sonic, or supersonic flow at the exit is now given.

A. Subsonic Flow at Exit

For subsonic flow it is assumed that the pressures on each side of the exit plane follow the steady state boundary condition of equal pressure. Since we have chosen the conditions which were on the right side of the diaphragm before rupture as the reference state, the pressure at the exit is given by $p_{E_L} = p_{E_R} = 1$ where the subscripts E_L and E_R refer to the left- and right-hand sides of the exit plane, respectively. The iteration procedure to calculate the flow properties at the exit is as follows:

(1) The velocity u_{E_L} is given by the previous iteration cycle, or for the first cycle is guessed.

(2) The entropy S_E at the exit is given by the conventional procedure as for a regular point (Section V, Paragraph A).

(3) Applying the boundary condition p = 1 at the exit results in the following equation for the speed of sound, we obtain

$$a_{E_L} = e^{\frac{\gamma-1}{2}(S_{E_L})},$$

which includes the conditions that the properties on the right side of the exit plane are the reference state and the reference entropy is zero.

(4) The value for the P characteristic P_{E_L} at the exit is found in the same manner as for a regular point. This value then furnishes a relation for the velocity:

$$u_{E_L} = P_{E_L} - \frac{2}{\gamma - 1} a_{E_L}.$$

Steps (2) through (4) are repeated until the change in u_{E_L} for subsequent iteration cycles is less than a prescribed tolerance.

B. Sonic Flow at Exit

The boundary condition to be satisfied for sonic flow at the exit is $u_{E_L} = a_{E_L}$. The trial-and-error procedure to calculate the sonic flow conditions at the exit is as follows:

(1) Assume that the velocity u_{E_L} has been given from the previous iteration cycle or for the first cycle has been guessed.

(2) Satisfy the boundary condition by setting $a_{E_L} = u_{E_L}$.

(3) Values for the entropy S_{E_L} and characteristic P_{E_L} are found by the standard procedure (Section V, Paragraph A).

(4) A new value for u_{E_L} is given by

$$u_{E_L} = \frac{\gamma - 1}{\gamma + 1} P_{E_L} .$$

Steps (2) through (4) are repeated until the change in u_{E_L} for subsequent iteration cycles is less than a prescribed tolerance.

C. Supersonic Flow at Exit

For supersonic outflow, the exit conditions can be calculated by the standard procedure since both P and Q characteristics reach the exit.

X. SOME PERTINENT DETAILS ABOUT THE CALCULATION PROCEDURE

A. Geometrical Aspects

Effects on the flow properties due to the change in cross section are felt through the term

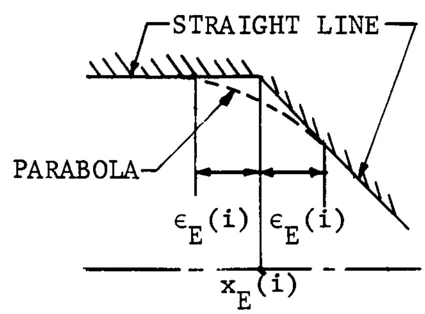
$$\frac{1}{A} \frac{dA}{dx} ,$$

which appears in both characteristic equations. Information concerning the geometry of the duct must be furnished to the computer program in a

form such that the term

$$\frac{1}{A}\frac{dA}{dx}$$

can be evaluated at any point x. The actual duct shape is approximated by a series of straight line segments for the inside diameter in the axial direction. To avoid discontinuities in the first derivatives of the area at the junction of the line segments, a parabola which connected the straight lines at a distance of ϵ on each side of the junction was used as shown in the illustration.



The computer program requires M input cards regarding the geometry for which each card contains an axial station $x_E^*(i)$ along with the corresponding diameter $d_E^*(i)$, and curve fit parameter $\epsilon_E^*(i)$, where $i = 1,M$. The left end of the supply tube $x^* = 0$ is referred to by $i = 1$, and the right end of the facility where $x^* = L_o$ is referred to by $i = M$. The remaining indices $i = 2,M-1$ refer to the junction of the straight line segments. The program automatically nondimensionalizes all distances by L_o.

B. Grid Control

To keep the characteristic grid at a sufficient density, it was necessary to establish some criteria to either introduce or take out points at each time step. It is seen, for instance, that in regions where there is sonic flow, additional points must be introduced, since in subsonic flow, the Q characteristics travel to the left, and in supersonic flow, they travel to the right in the wave diagram (see Figure 2).

Whether or not points should be introduced was determined by prescribing an upper limit in the change of Q between two consecutive points and an upper limit in the allowable distance between them. Let these limits be referred to as ΔQ_{max} and Δx_{max} and consider that the solution has been completed up to a point j on a line $t = t_1$. The following quantities are then calculated:

$$\Delta Q = Q(j) - Q(j-1)$$

$$\Delta x = x(j) - x(j-1).$$

If either ΔQ or Δx is larger than the prescribed upper limits for these quantities, any number from one to three additional points or Q waves are introduced between x(j) and x(j-1). These additional points are calculated by the method of characteristics in a special subroutine called POINTADD of the computer program. The maximum allowable difference in Q between two adjacent points ΔQ_{max} is a required input (TOLQ) of the computer program. Values of TOLQ = .05 for the subsonic cases and TOLQ = .2 for the supersonic cases were found to give good results. The upper limit for the distance between any two points Δx_{max} is automatically determined by the computer program. The value of Δx_{max} is not constant throughout the duct since, in sections where there are area changes, a much denser distribution of points is necessary than in straight sections where there are no area changes. The program also takes out points automatically when the grid becomes too dense.

By running repeated cases for several time increments, it was found that Δt = .001 for subsonic and Δt = .002 for supersonic cases gave satisfactory results. What is meant by subsonic and supersonic cases as referred to above pertains to the state of the flow in the test section after the flow is established. The distance between the points in the axial direction was found to be much more critical than the time interval in the sections where there were large area changes.

C. Restart Procedure

Since each case took a fairly large amount of time on the CDC 3200 computer, a restart procedure was incorporated in the program. Each time on the computer, a prescribed number of time intervals are computed and the distribution of points and their pertinent properties for the last calculated time are punched on computer cards, which are then used as input for the next run. The preparation of input for the program is explained in detail in Appendix B.

XI. DISCUSSION OF THE RESULTS

The calculation procedures described in this report for the one-dimensional flow through the pressure-tube tunnel were programmed for the CDC 3200 computer. Calculations were performed for the anticipated range of Mach numbers for the facility. Figure 1A presents the aerodynamic boundaries for the full-scaled facility with the Mach 2 nozzle installed. The interchangeable nozzle sections for the supersonic cases were designed from the results of a two-dimensional method of characteristics satisfying the condition of uniform flow at the nozzle exit plane. The spool length satisfies the condition that the length of the nozzle and spool equaled 138 inches for each case. The distance from the

diaphragm to the nozzle throat was approximately 29 percent longer for $M_{TEST} = 3.5$ and 5.0 than the corresponding length for $M_{TEST} = 1.4$, 1.7, and 2.04. Figure 1B presents the aerodynamic boundaries of the tunnel with the sonic nozzle installed. This configuration was used for the sonic and subsonic cases that were calculated. The subsonic Mach numbers in the test section were attained by choking the flow downstream of the test section by the use of choking flaps as shown in Figure 1B. Also shown in Figures 1A and 1B are imaginary or effective boundaries which were assumed to approximately account for blockage by the model support and diaphragm cutter mechanisms.

The following initial and reference conditions were assumed for all of the cases calculated:

(1) The gas on each side of the diaphragm is air whose ratio of specific heats is $\gamma = 1.4$.

(2) The temperature of the air on each side of the diaphragm is $T_1^* = T_2^* = 295.57$ °K.

(3) The properties of the air on the right side of the diaphragm before rupture are:

$$a_1^* = 1130 \text{ ft/sec}$$

$$p_1^* = 14.7 \text{ lb/in}^2 = 2116.8 \text{ lb/ft}^2$$

$$S_1 = 0.$$

(4) The reference conditions are the state of the air on the right side of the diaphragm before rupture.

(5) The reference length is the total length of the facility:

$$L_o = 325 \text{ ft.}$$

Figure 2 presents the wave diagram for the case in which the Mach number in the test section during the period of steady flow (referred to as M_{TEST}) was two. Briefly recapping what occurs during the early development of the flow, we can see the centered expansion fan, which is bounded on the left by the head-wave and on the right by the tail-wave, that originates at the diaphragm location at the instant the diaphragm ruptures. The expansion waves that make up this centered expansion fan accelerates the air which was initially on the left side of the diaphragm so that it starts to flow out through the facility. Also shown is the P shock that was formed when the diaphragm was ruptured. The P shock accelerates the still air through which it passes

as it travels out of the duct. The path of the contact surface, i.e., the interface that divides the two volumes of air of different entropy that were initially in contact with each side of the diaphragm before rupture, is shown as it travels out of the duct. Between the tail-wave of the initial centered expansion fan and the contact surface, two Q shocks are formed which eventually interact with each other. Created at this interaction are a shock stronger than either of the original two, a contact surface, and a weak expansion wave. As soon as the sonic Mach number is reached at the nozzle throat, no more expansion waves can pass through, and therefore only an expansion fan of finite width can travel to the left through the supply tube. A short time after the flow in the throat chokes, a Q shock is formed just downstream of the nozzle throat and slowly travels through the remainder of the nozzle and the test section. A steady flow is established behind this Q shock, and the start time is therefore defined as the time when the Q shock passes out of the test section. Some of the Q and P characteristic curves are traced in the wave diagram presented in Figure 2. It is seen that the paths of the P characteristic curves are fairly uniform throughout the wave diagram, but the paths of the Q characteristics do not follow any systematic patterns in the early stages of the flow. After establishment of steady flow, both characteristics follow a uniform pattern; i.e., at each x station the slopes of both characteristics are constant in time after establishment of steady flow. At any time, we can recognize the flow regime at any position in the facility by observing the directions of the Q characteristics. Subsonic Mach numbers are characterized by the Q characteristic curves traveling to the left, sonic when traveling in the vertical direction, and supersonic when traveling to the right.

Figures 3 through 6 present the wave diagrams for test Mach numbers 1.4, 1.7, 3.5, and 5.0. The wave diagrams for all supersonic cases are similar to Figure 2, and therefore the Q and P characteristic net is not shown in Figures 3 through 6, because to enable one to draw the characteristic curves requires a print-out at each time step by the computer program. This print-out takes much more computer time and results in volumes of paper.

Figure 7 plots the static pressure as a function of time at a station in the test section for the supersonic range of test section Mach numbers calculated. The drop in pressure to a constant value after the shock passes this station thereby establishes the starting time defined as the beginning of the period of steady flow.

For the subsonic cases, the flow is choked downstream of the test section by choking flaps which are approximated by assumed boundaries that account for the flaps and blockage due to the model support and diaphragm cutter mechanism. Cases were calculated in which subsonic and sonic Mach numbers of .515, .7, and 1.0 resulted in the test section during the period of steady flow conditions. For the subsonic, as in

the supersonic cases, only an expansion fan of finite width can pass through the throat since no more expansion waves can pass through the throat after choking occurs. Steady flow is established in the test section only after the highly nonlinear effects of area changes on the slopes of the P and Q characteristic variables at each x station upstream of the throat have died out with time. This effect is illustrated in Figure 8, which shows the pressure as a function of time at an axial station in the test section. The starting times for the subsonic and sonic cases are defined as the times when steady state conditions are established throughout the test section.

Starting times for all of the cases that were calculated are plotted in figure 9 as a function of the test Mach number. It is seen that the subsonic and sonic Mach numbers require a longer start time, since the steadying of the flow properties in the test section is a gradual process whereby, for the supersonic cases, a strong shock passes through the test section that instantly establishes a steady flow behind it. The starting times for the supersonic cases depend, however, on the speed at which the shock travels through the nozzle and test section. This speed is governed by the local flow conditions upstream of the shock and the strength of the shock.

Figure 10 presents the calculated Reynolds number per foot in the test section during the period of steady flow for the range of test Mach numbers and for the maximum charge pressure of 48.64 atm assumed for the facility. It is seen that the highest test Reynolds number of approximately 2×10^8 per foot for the facility will occur in the vicinity of a test Mach number of 1.4. Figure 10 shows one case which was calculated for a test Mach number of 2.04 in which the charge pressure was assumed to be 18.03 atm. This reduction in charge pressure resulted in approximately a linear reduction in test Reynolds number for this particular Mach number. The effect on starting time by this reduction in charge pressure was found to be negligible, however.

The static pressure in the test section during the period of steady flow is presented as a function of Mach number in Figure 11. The expected trend of decreasing pressure with increasing Mach number is shown.

To check the effects of the settling chamber on the start time, one case for $M_{TEST} = 2.04$ was calculated in which the settling chamber was removed. A comparison of these results with the corresponding case with a settling chamber showed a negligible difference in the start time.

The preliminary analytical results that have been calculated by this procedure will be compared with experimental results when the facility is completed and measured data made available. Future analytical investigations to include the effects of boundary layer and mass removal of gas through the walls in the test section for transonic Mach numbers are planned.

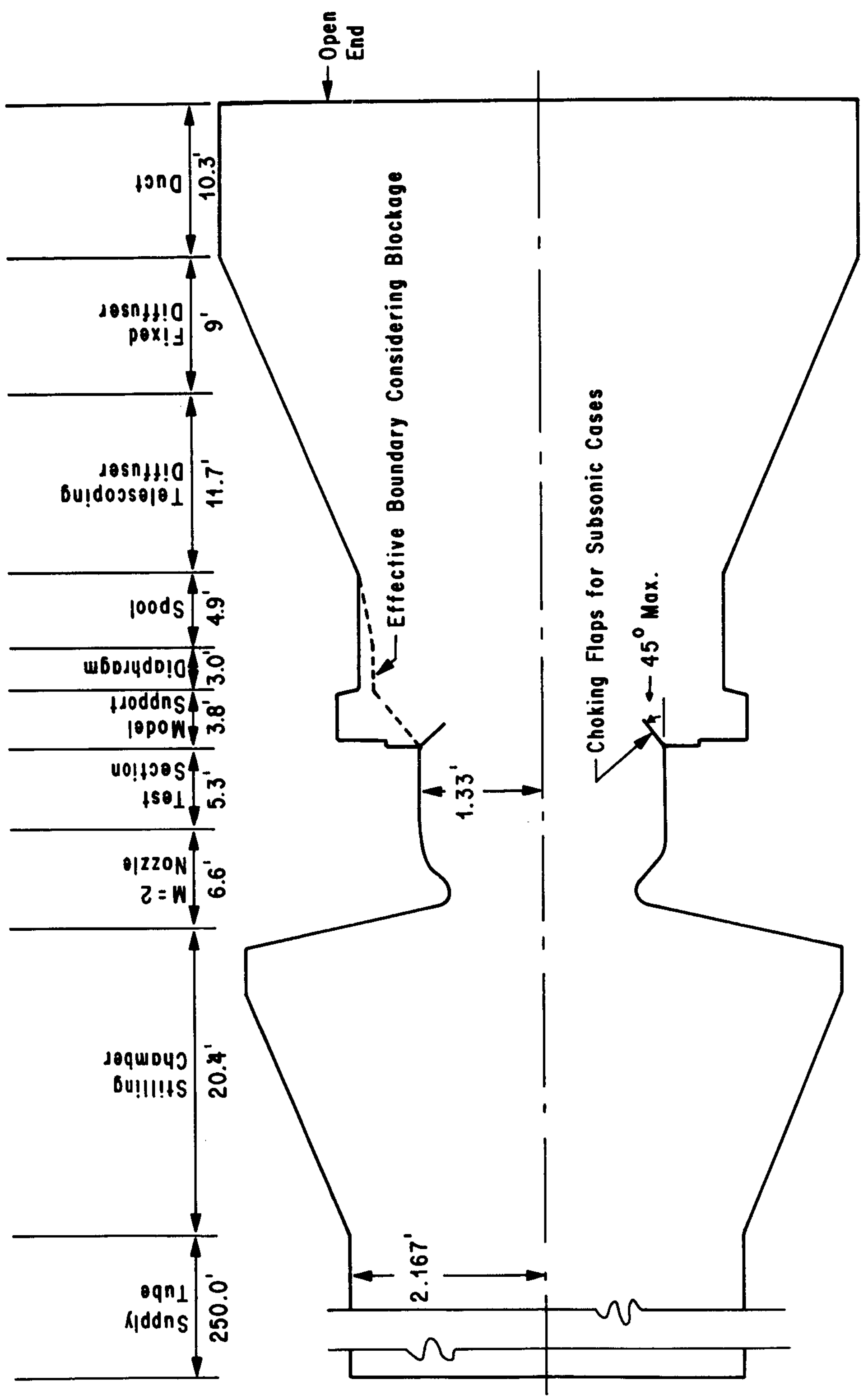


FIG. 1A AERODYNAMIC BOUNDARIES OF FACILITY WITH THE MACH 2 NOZZLE

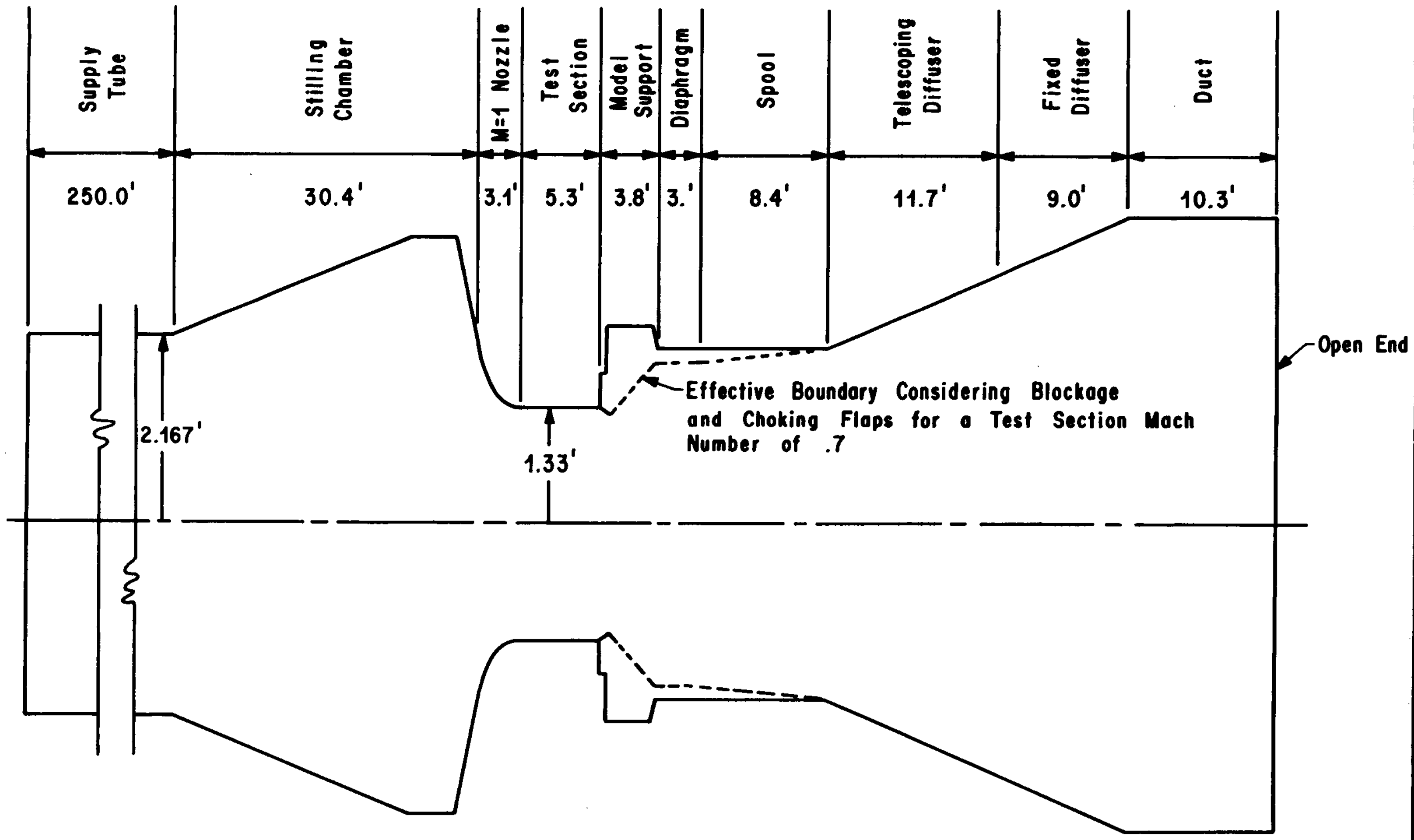


FIG. 1B. AERODYNAMIC BOUNDARIES OF FACILITY WITH THE MACH 1 NOZZLE

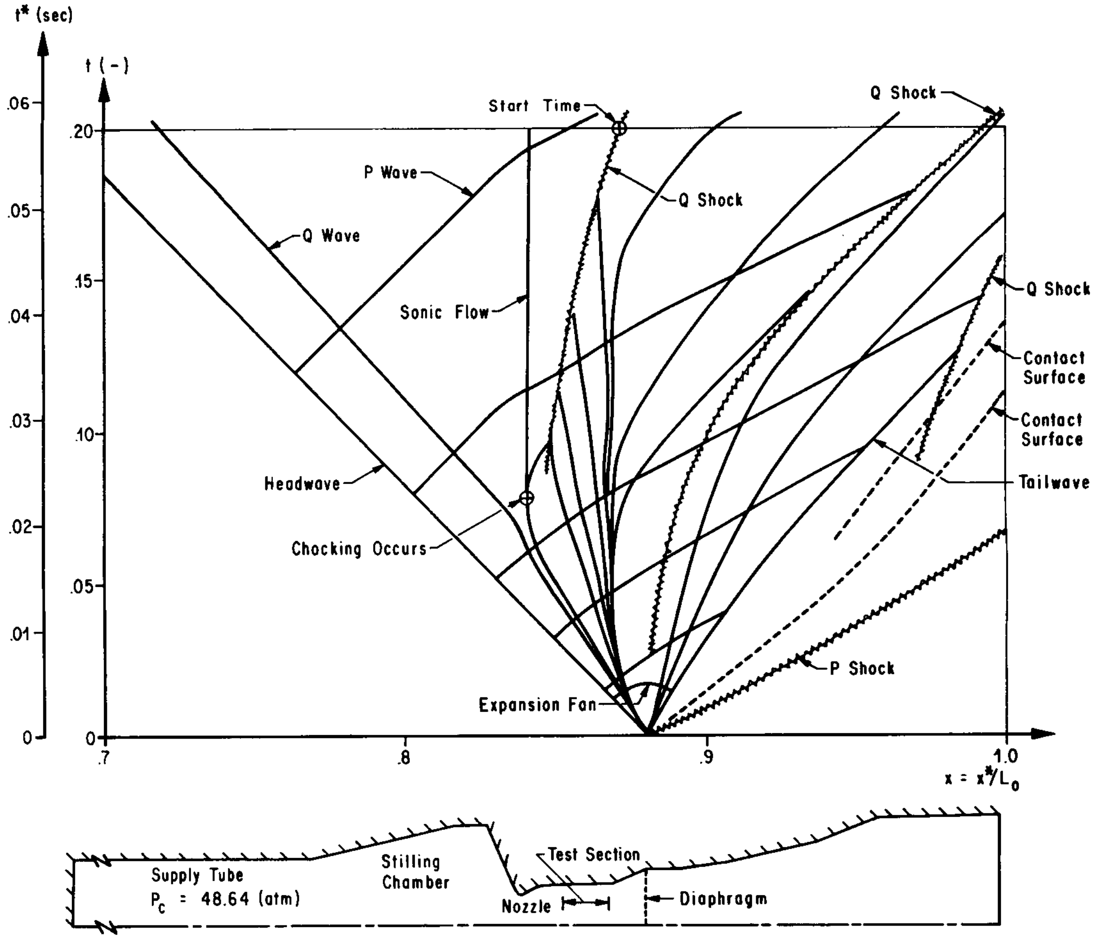


FIG. 2. WAVE DIAGRAM FOR M_{TEST} = 2.0

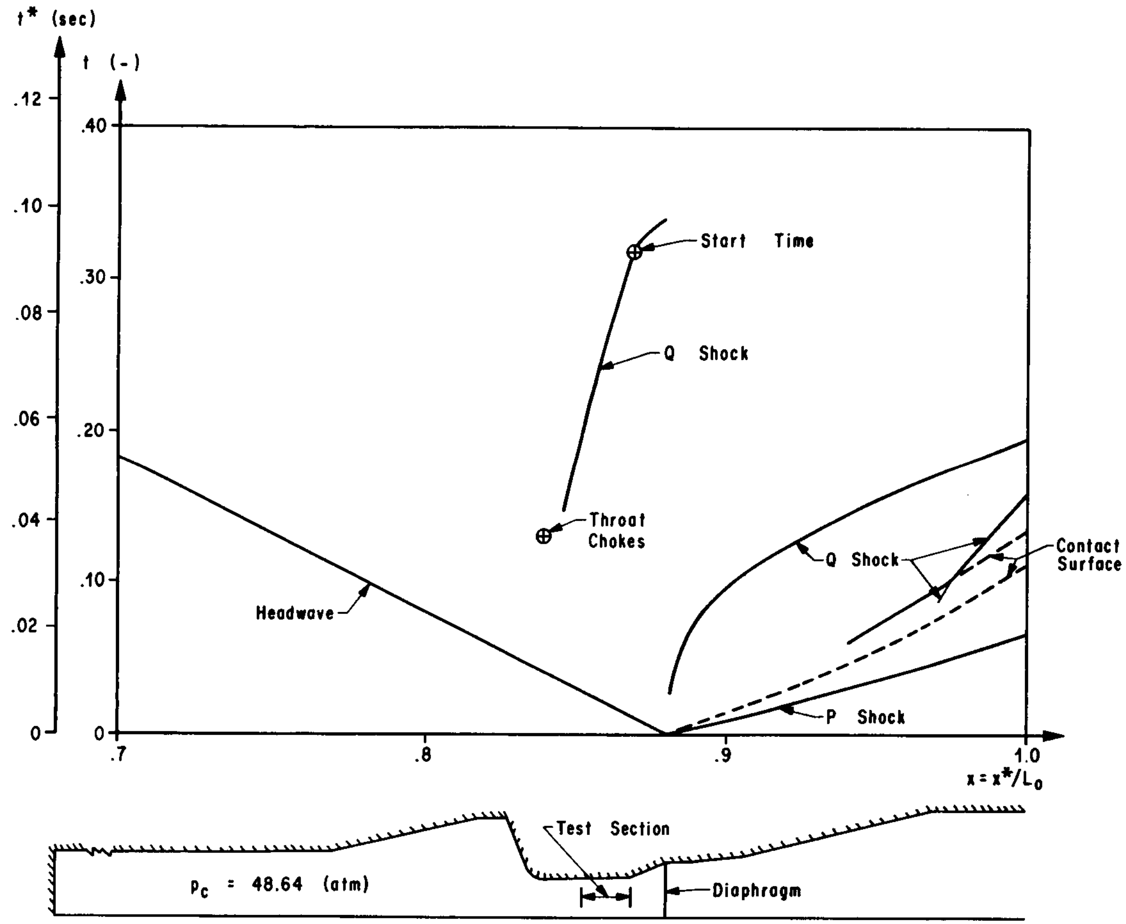


FIG. 3. WAVE DIAGRAM FOR M_{TEST} = 1.4

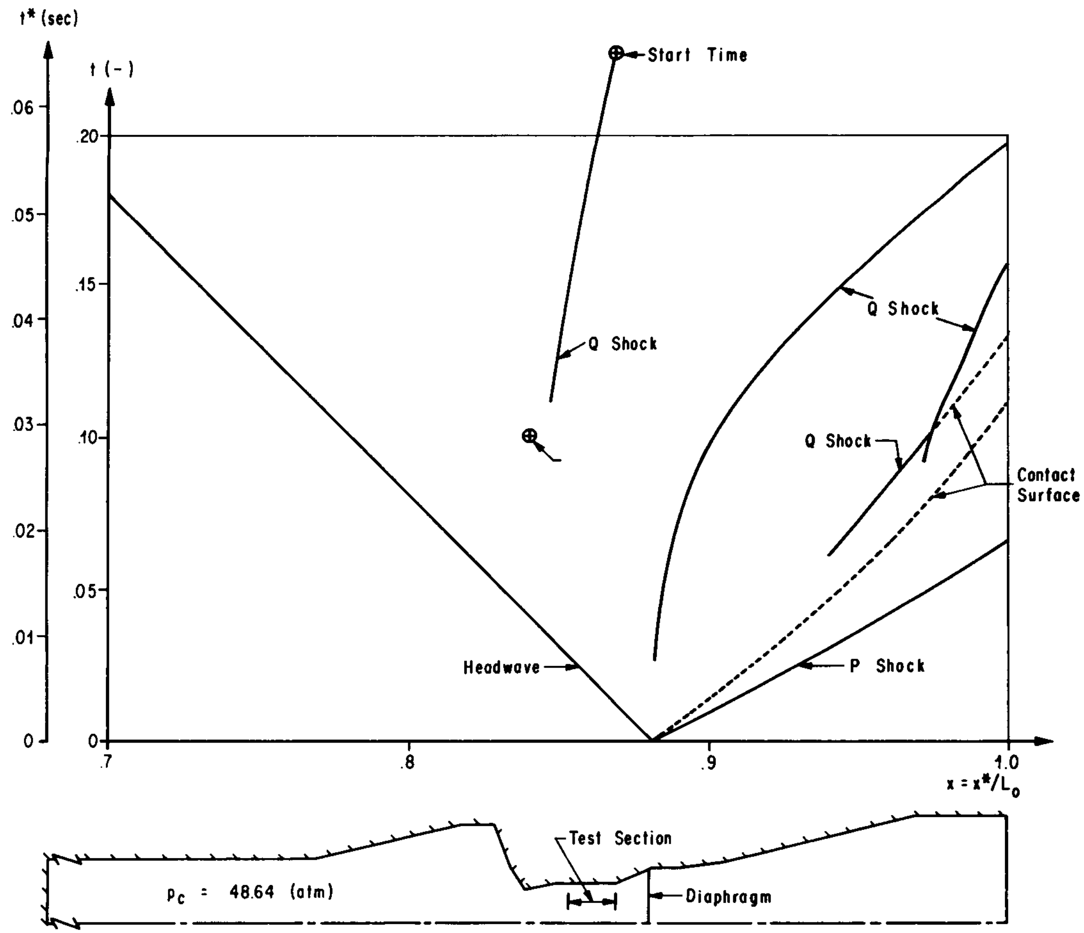


FIG. 4. WAVE DIAGRAM FOR M_{TEST} = 1.7

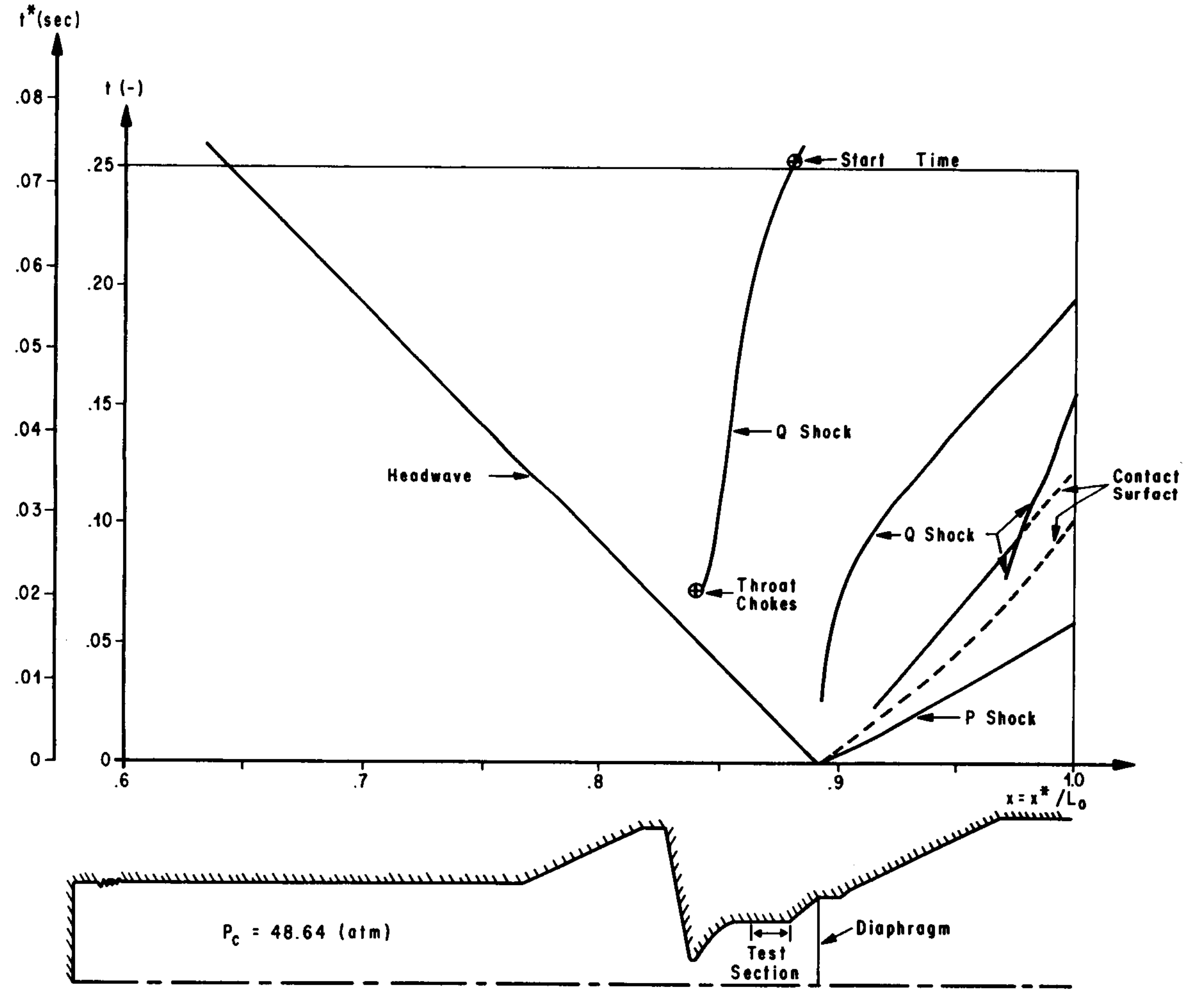


FIG. 5. WAVE DIAGRAM FOR M_{TEST} = 3.5

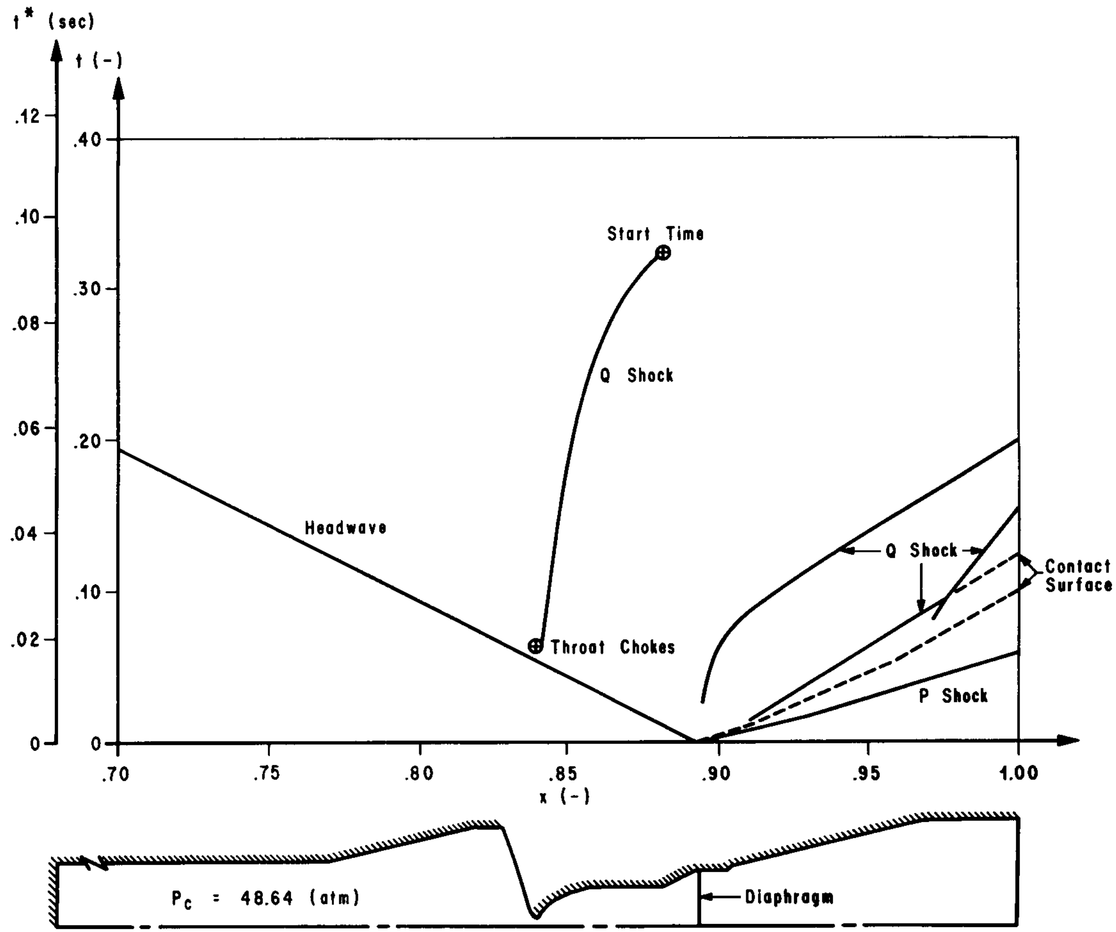


FIG. 6. WAVE DIAGRAM FOR M_{TEST} = 5.0

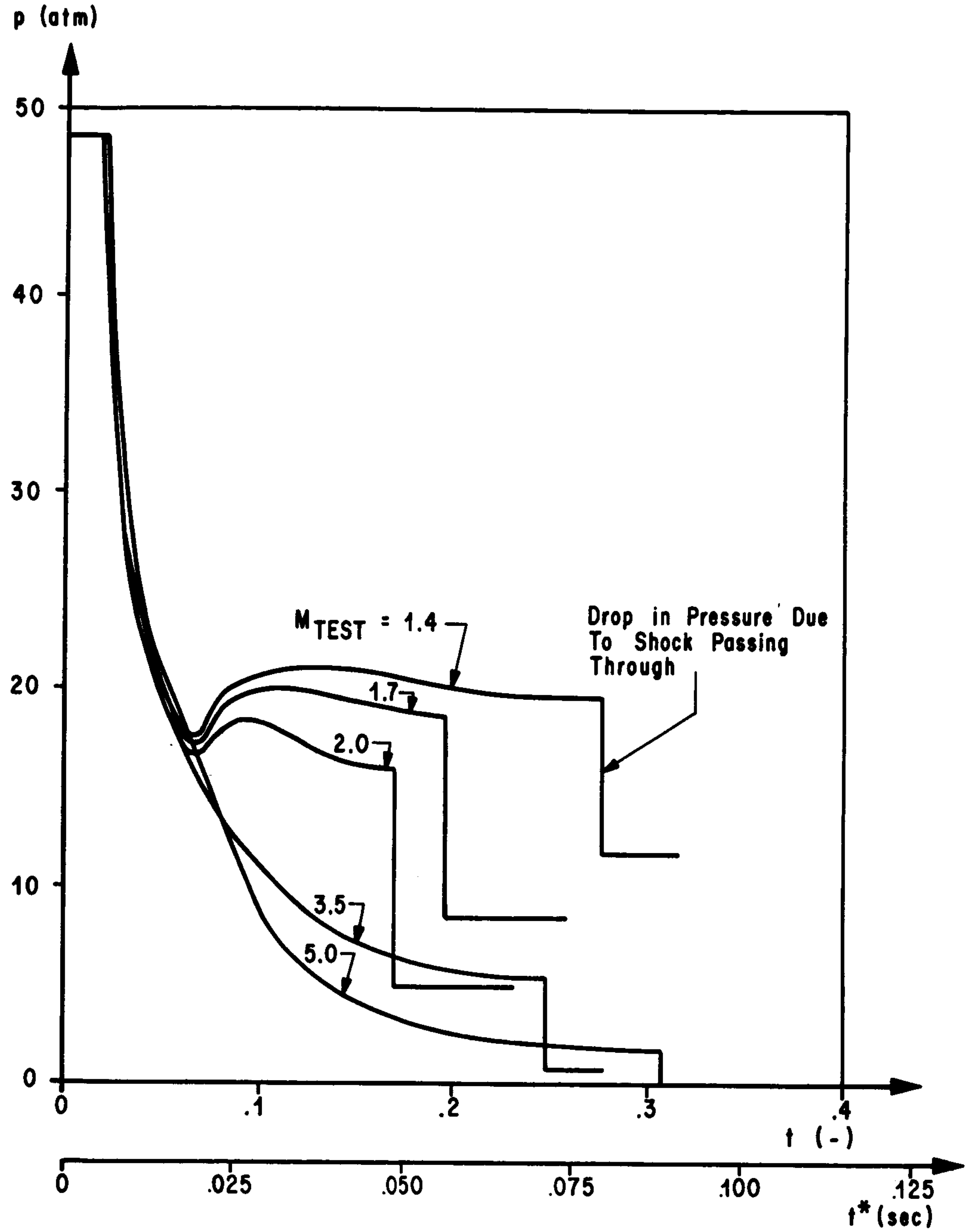


FIG. 7. PRESSURE AS A FUNCTION OF TIME AT A STATION IN THE TEST SECTION FOR SUPERSONIC TEST MACH NUMBERS

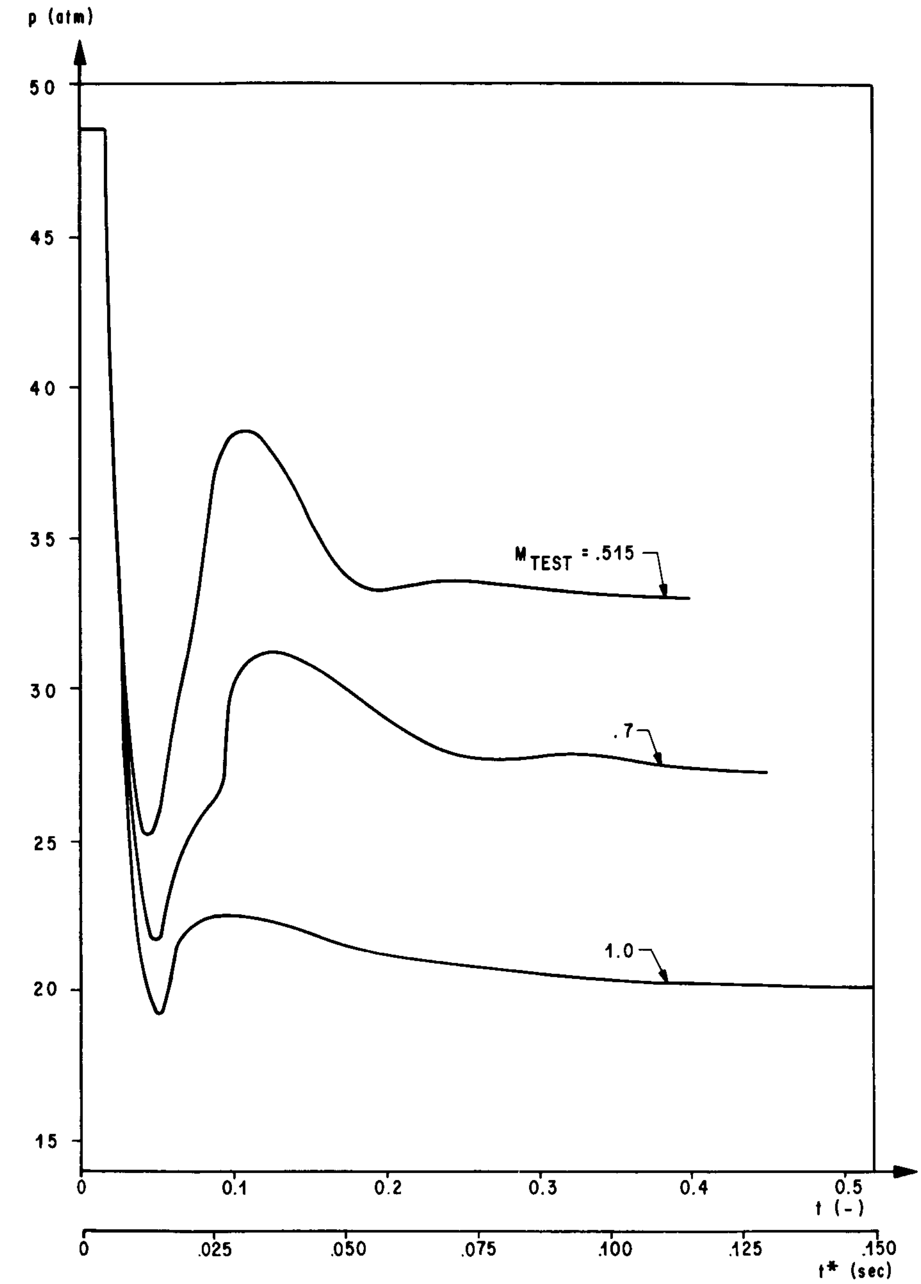


FIG. 8. PRESSURE AS A FUNCTION OF TIME
AT A STATION IN THE TEST SECTION
FOR SUBSONIC AND SONIC TEST MACH NUMBERS

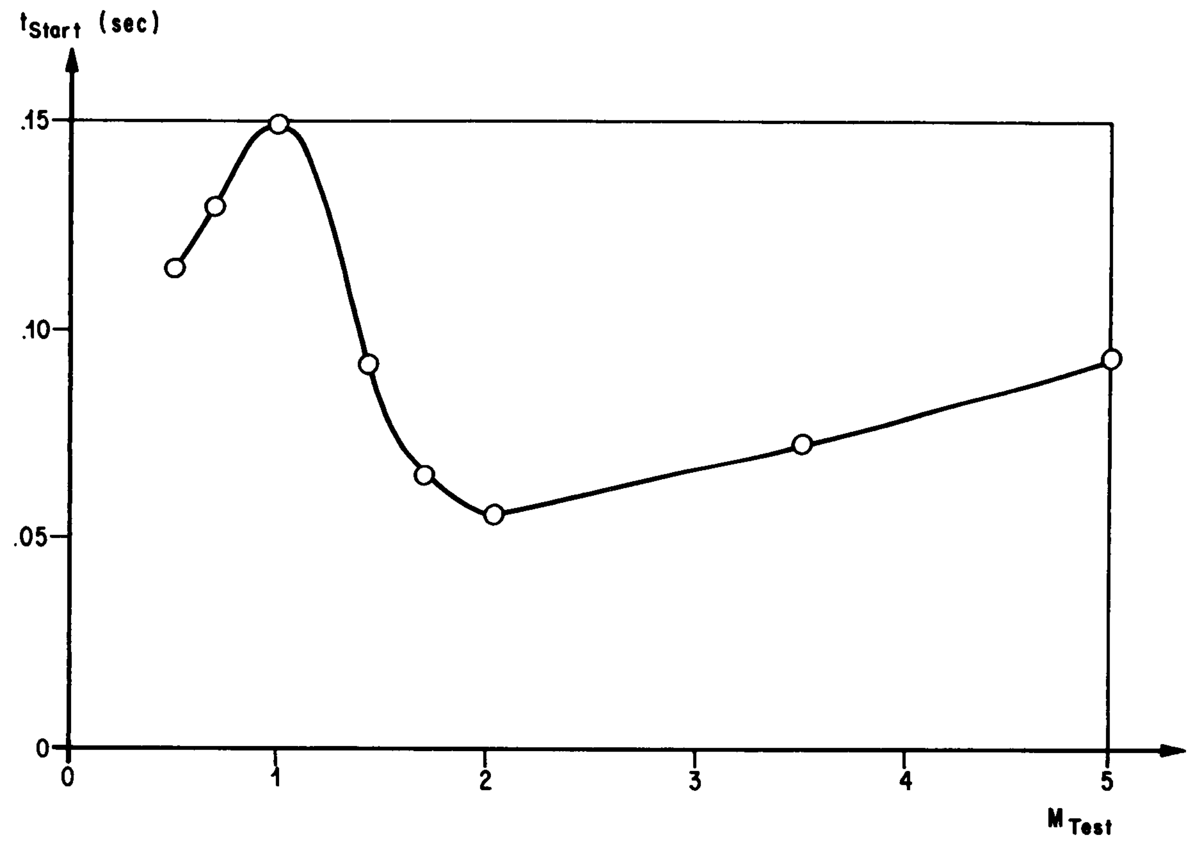


FIG. 9. START TIME AS A FUNCTION OF TEST MACH NUMBER

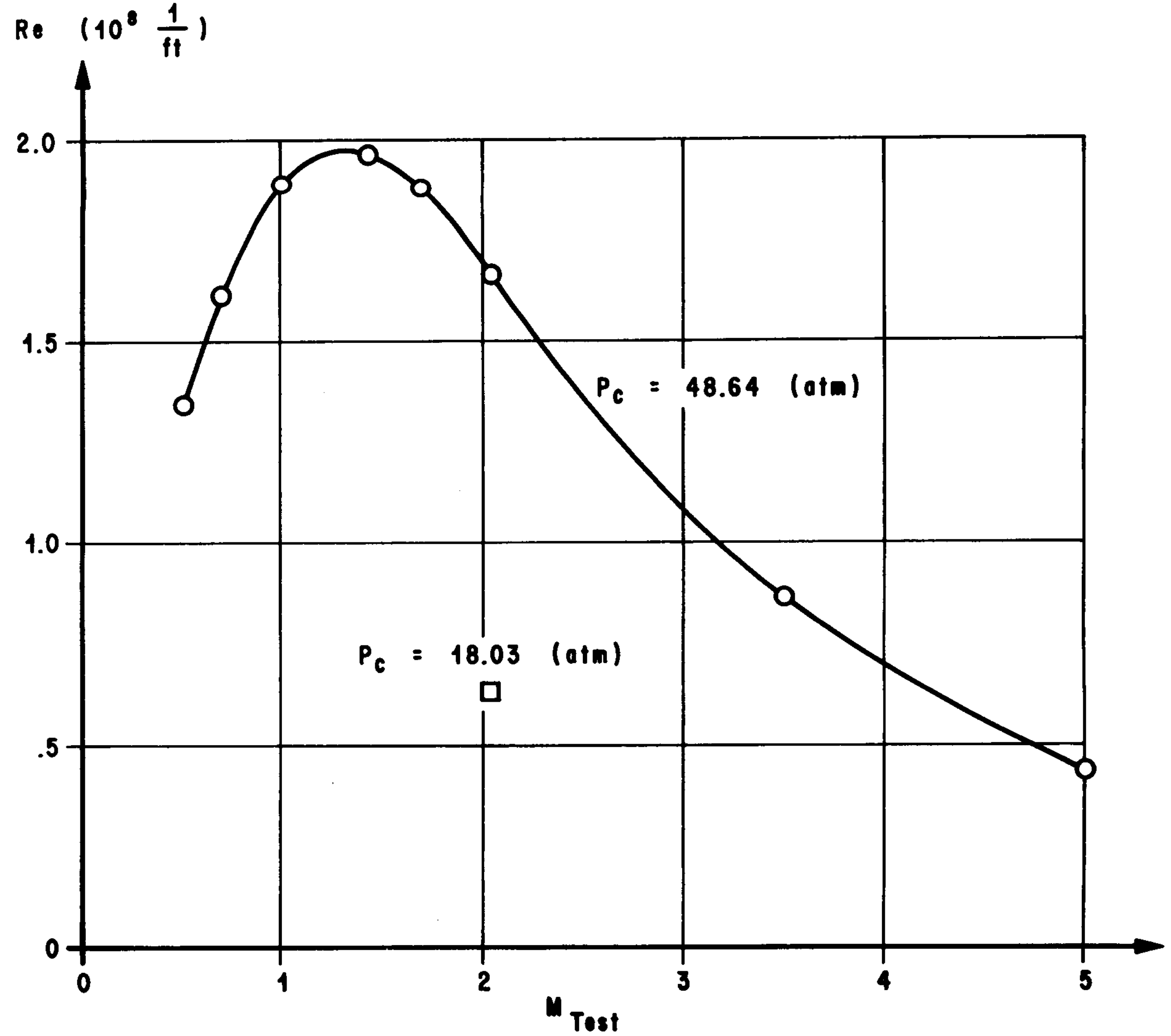


FIG. 10. TEST REYNOLDS NUMBER AS A FUNCTION OF TEST MACH NUMBER

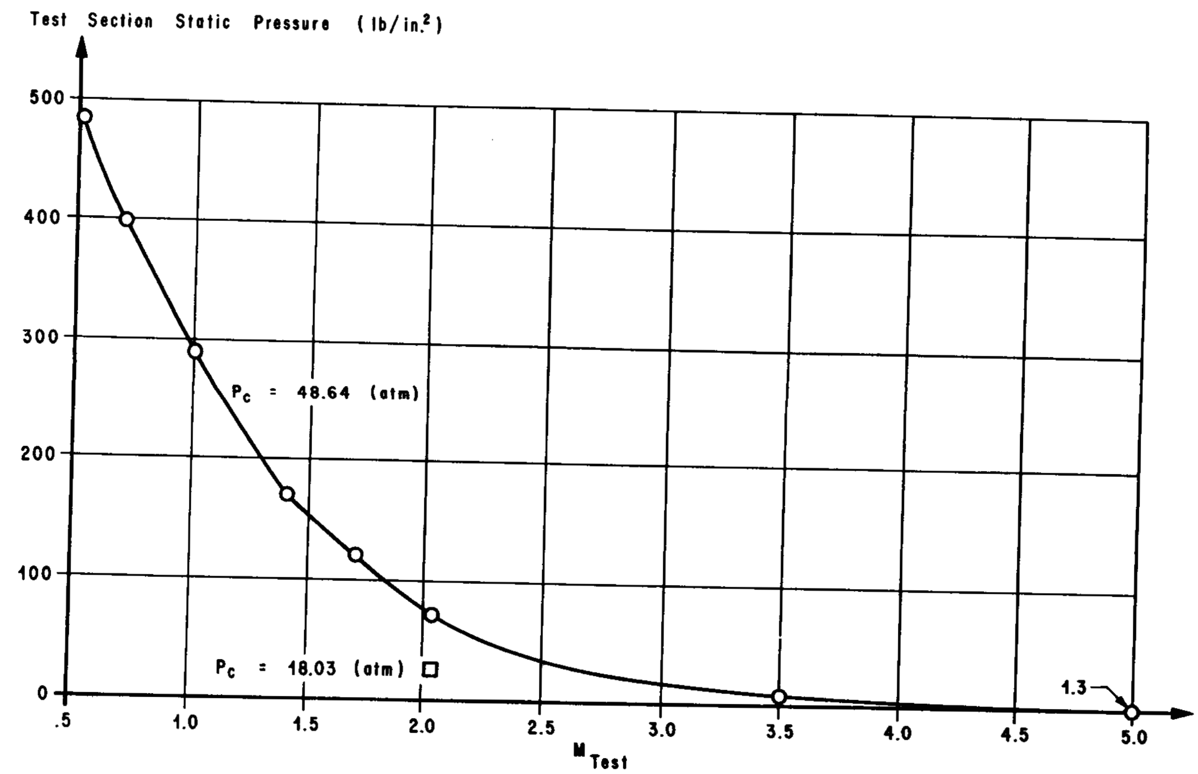


FIG. 11. STATIC PRESSURE AS A FUNCTION OF MACH NUMBER IN THE TEST SECTION DURING STEADY FLOW PERIOD

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APPENDIX A

Derivation of the Characteristic Equations

The basic differential equations (1) through (4) can be written in nondimensional form (see Definition of Symbols) as:

Continuity

$$\frac{1}{A}\frac{\partial(\rho A)}{\partial t} + \frac{1}{A}\frac{\partial(\rho u A)}{\partial x} + \psi = 0. \tag{A-1}$$

Momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{a^2}{\gamma}\frac{\partial(\ln p)}{\partial x} + f. \tag{A-2}$$

Equation of State

$$a^2 = \frac{p}{\rho} = T. \tag{A-3}$$

First Law of Thermodynamics

$$S - S_o = \frac{1}{\gamma - 1}\ln\frac{p}{\rho} - \frac{1}{\gamma}\ln p. \tag{A-4}$$

Some useful relations that result from the combining of equations (A-3) and (A-4) are

$$p/\rho^{\gamma} = e^{\gamma(\gamma-1)(S-S_o)} \tag{A-5a}$$

$$p = a^{2\gamma/(\gamma-1)}\ e^{-\gamma(S-S_o)} \tag{A-5b}$$

$$\rho = a^{2/(\gamma-1)}\ e^{-\gamma(S-S_o)}. \tag{A-5c}$$

Due to the relations (A-5a) through (A-5c), the continuity equation (A-1) can be expressed as

$$\frac{2}{\gamma - 1} \frac{\partial a}{\partial t} + \frac{2}{\gamma - 1} u \frac{\partial a}{\partial x} + a \frac{\partial u}{\partial x} = - au \frac{\partial(\ln A)}{\partial x} - a \frac{\partial(\ln A)}{\partial t}$$

$$+ \gamma a \left(\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x}\right) - \psi a^{(\gamma-3)/(\gamma-1)} e^{\gamma(S-S_o)}, \tag{A-6}$$

and the momentum equation as

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{2}{\gamma - 1} a \frac{\partial a}{\partial x} = a^2 \frac{\partial S}{\partial x} + f. \tag{A-7}$$

Adding equation (A-7) to equation (A-6) results in the following equation:

$$\frac{\partial P}{\partial t} + (u + a) \frac{\partial P}{\partial x} = -au \frac{\partial(\ln A)}{\partial x} - a \frac{\partial(\ln A)}{\partial t} + \gamma a \left(\frac{DS}{Dt} + \frac{a}{\gamma} \frac{\partial S}{\partial x}\right)$$

$$- \psi a^{(\gamma-3)/(\gamma-1)} e^{\gamma(S-S_o)} + f, \tag{A-8}$$

where

$$P = \frac{2}{\gamma - 1} a + u. \tag{A-9}$$

The substantial derivative in the x,t-plane is defined as

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} . \tag{A-10}$$

Let the differential operator

$$\frac{\delta_+}{\delta t} = \frac{\partial}{\partial t} + (u + a) \frac{\partial}{\partial x} \tag{A-11}$$

represent the change of a parameter in the direction

$$\frac{dx}{dt} = u + a \tag{A-12}$$

in the x,t-plane. Equation (A-8) can then be written as

$$\frac{\delta_+ P}{\delta t} = - au \frac{\partial(\ln A)}{\partial x} - a \frac{\partial(\ln A)}{\partial t} + a(\gamma - 1) \frac{DS}{Dt} + a \frac{\delta_+ S}{\delta t}$$

$$- \psi a^{(\gamma-3)/(\gamma-1)} e^{\gamma(S-S_o)} + f. \tag{A-13}$$

Subtracting equation (A-7) from (A-6) yields

$$\frac{\partial Q}{\partial t} + (u - a) \frac{\partial Q}{\partial x} = -au \frac{\partial(\ln A)}{\partial x} - a \frac{\partial(\ln A)}{\partial t} + \gamma a \left(\frac{DS}{Dt} - \frac{a}{\gamma} \frac{\partial S}{\partial x}\right)$$

$$- \psi a^{(\gamma-3)/(\gamma-1)} e^{\gamma(S-S_o)} - f, \tag{A-14}$$

where

$$Q = \frac{2}{\gamma - 1} a - u. \tag{A-15}$$

Let the differential operator

$$\frac{\delta_-}{\delta t} = \frac{\partial}{\partial t} + (u - a) \frac{\partial}{\partial x} \tag{A-16}$$

represent the change of a parameter in a direction

$$\frac{dx}{dt} = u - a \tag{A-17}$$

in the x,t-plane. Equation (A-14) can then be written as

$$\frac{\delta_- Q}{\delta t} = -au \frac{\partial(\ln A)}{\partial x} - a \frac{\partial(\ln A)}{\partial t} + a(\gamma - 1) \frac{DS}{Dt} + a \frac{\delta_- S}{\delta t}$$

$$- \psi a^{(\gamma-3)/(\gamma-1)} e^{\gamma(S-S_0)} - f. \qquad \text{(A-18)}$$

A third relation is required to complete the system for the three dependent variables a, u, and S. Given by the entropy condition which must be prescribed for any problem, this relation is given here in a general form as

$$\frac{DS}{Dt} = F(a,u,S,x,t). \qquad \text{(A-19)}$$

Equations (A-13), (A-18), and (A-19) form the system of equations that must be solved for the three dependent variables u, a, and S. These equations are solved by a step-by-step procedure along the characteristic curves P, Q, and S in the x,t-plane. The details of solving these equations are presented in other sections of this report.

APPENDIX B

The Fortran Program and Its Input Data

1. Preparation of the Input Data for the Computer Program

The computer program and the subroutines employed by the program are written in Fortran IV language for the CDC 3200 computer in Part 2 of this Appendix. To run the program, the following input cards must be prepared:

Card No.	Columns	Fortran Symbol	Format	Description
1	1-14	TOL	E14.8	Tolerance for which all solutions calculated by an iteration procedure is satisfied.
	15-28	TOLQ	E14.8	Maximum difference Q_{MAX} between the values of the characteristic Q for neighboring Q waves (see Section X, Paragraph B).
	29-42	P2P1	E14.8	Ratio of the pressure on the left-hand side of the diaphragm to that on the right before rupture (p_2^*/p_1^*).
	43-56	A2	E14.8	Nondimensional speed of sound on left side of diaphragm before rupture; see equation (33).
2	1-14	G	E14.8	Ratio of specific heat (γ). The program assumes that the gases on both sides of the diaphragm have the same value for γ.
	15-28	XREF	E14.8	Total facility length (L_o) by which all distances are nondimensionalized.

Card No.	Columns	Fortran Symbol	Format	Description
	29-42	XD	E14.8	Diaphragm location (x_D^*).
	43-56	XTEST	E14.8	Axial location (x_{TEST}^*) in test section for which pressure and Mach number are determined and printed out.
	57-59	M	I3	Number of cards used in describing tunnel geometry (see Section X, Paragraph A).
3 to M+2	1-13	XE	E14.8	Axial station (x_E^* for which diameter and curve parameter are given.
	15-28	DE	E14.8	Diameter (d_E^*) of the axially symmetric duct at x_E^*.
	29-42	EPS	E14.8	Curve fit parameter (ϵ_E^*) at x_E^* (see Section X, Paragraph A).
M+3	1-14	DT	E14.8	Nondimensional time interval.
	15-17	NUMT	I3	Number time steps to be located.
	18-20	NPRNT	I3	Printout frequency.
M+4	1-15	T	E15.8	Nondimensional time.

For $T > 0$, the program incorporates a restart procedure that requires information from the last calculated time in order to resume calculations. This information is contained in a number of cards that have been automatically prepared at the termination of the previous run. Therefore, for $T > 0$, the input data, beginning with card #M+4 plus the remaining required input, is automatically prepared at the termination of the previous run.

For $T = 0$, card #M+4 is the last input data card.

2. The Fortran Program and Subroutines

```
      PROGRAM DECK 3
C     CONSTANT TIME INTERVAL METHOD
      DIMENSION ZT(10),C1(1),XQ(1),QQ(1)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),ICONTACT(10)
      READ(60,88)TOL,TOLQ,P2P1,A2
   88 FORMAT(4E14.8)
      READ(60,90)G,XREF,XD,XTEST,M
   90 FORMAT(4E14.8,I3)
      XTEST=XTEST/XREF
      C=2.*G/(G-1.)
      JPRNT=1
C     G IS THE RATIO OF SPECIFIC HEATS
C     XREF IS THE REFERENCE LENGTH WITH DIMENSIONS (TOTAL LENGTH)
C     XD IS THE DIAPHRAGM LOCATION WITH DIMENSIONS
C     M IS THE TOTAL NUMBER OF GEOMETRIC POINTS READ IN
      WRITE(61,89)
   89 FORMAT(//3X,29HCONSTANT TIME INTERVAL METHOD/)
      WRITE(61,95)XREF
   95 FORMAT(3X,2HL=,E15.8,1X,18H(REFERENCE LENGTH))
      XD=XD/XREF
      WRITE(61,94)XD
   94 FORMAT(/3X,5HXD/L=,E15.8,1X,19H(DIAPHRAGM STATION)////)
      WRITE(61,300)
  300 FORMAT(34X,15HTUNNEL GEOMETRY/)
      WRITE(61,301)
  301 FORMAT(4X,1HI,6X,5HXE(I),7X,10HXE(I)/XREF,8X,5HDE(I),10X,6HEPS(I)
     16X,11HEPS(I)/XREF/)
      DO 80 I=1,M
      READ(60,100)XE(I),DE(I),EPS(I)
  100 FORMAT(3E14.8)
      XEP=XE(I)
      EPSP=EPS(I)
      XE(I)=XE(I)/XREF
      EPS(I)=EPS(I)/XREF
   80 WRITE(61,302)I,XEP,XE(I),DE(I),EPSP,EPS(I)
  302 FORMAT(3X,I3,5E15.8)
C     XE(I) IS VALUE OF X (WITH DIMENSIONS) AT STATION I
C     DE(I) IS DIAMETER (WITH DIMENSIONS) AT XE(I)
C     EPS(I) IS SMALL INCREMENT ON EACH SIDE OF XE(I) FOR WHICH THE
C              CUBIC CURVE FIT FOR THE DIAMETER IS USED
      XTL=XE(2)-EPS(2)
      DO 315 I=2,M
      IF(XTEST-XE(I))316,316,315
  315 CONTINUE
  316 ITESTR=I
      ITESTL=I-1
      IR1=ITESTR+1
      DO 398 IR=IR1,M
```

```
      IF(DE(IR)-DE(IR-1))396,398,399
 398  CONTINUE
 399  XTP=(XE(IR)+XE(IR-1))/2.
      DXIMAX=(XE(ITESTR)-XE(ITESTL))/20.
      DXOMAX=.015
      DXIMIN=DXIMAX/3.
      DXOMIN=DXOMAX/2.
      DQIMAX=TOLQ
      DQIMIN=TOLQ/3.
      DQOMAX=.15
      DQOMIN=DQOMAX/2.
      CALL GEOM(M,XE,DE,EPS,YC)
      WRITE(61,303)
  303 FORMAT(///5X,53HCOEFFICIENTS FOR PARABOLIC CURVE FIT BETWEEN SECTI
     1ONS)
      WRITE(61,304)
  304 FORMAT(3X,19HD=C1+C2*SI+C3*SI**2)
      WRITE(61,305)
  305 FORMAT(3X,66HSI=(X-(XE(I)-EPS(I)))/XREF AND XE(I)-EE(I) .LE. X .LE
     1. XE(I)+EE(I)/)
      WRITE(61,306)
  306 FORMAT(3X,1HI,5X,2HC1,13X,2HC2,13X,2HC3)
      M1=M-1
      DO 81 I=2,M1
   81 WRITE(61,307)I,YC(I,1),YC(I,2),YC(I,3)
  307 FORMAT(3X,I3,4E15.8)
      A1=1.
      U1=0.
      U2=0.
      P1=2.*A1/(G-1.)+U1
      S1=0.
      P2=2.*A2/(G-1.)+U2
      Q2=2.*A2/(G-1.)-U2
      READ(60,98)DT,NUMT,NPRNT
   98 FORMAT(E14.8,2I3)
C     DT IS THE TIME INTERVAL
C     NUMT IS THE NUMBER OF T WAVES TO COMPUTE
C     NPRNT IS THE NUMBER OF T WAVES BETWEEN PRINT OUT
      WRITE(61,200)G
  200 FORMAT(//3X,2HG=,E15.8,2X,7H(GAMMA))
      WRITE(61,201)TOLQ
  201 FORMAT(3X,5HTOLQ=,E15.8)
      WRITE(61,202)P2P1
  202 FORMAT(3X,6HP2/P1=,E15.8)
      WRITE(61,203)DT
  203 FORMAT(3X,3HDT=,E15.8)
      WRITE(61,110)
  110 FORMAT(1H1)
      READ(60,86)T
   86 FORMAT(E15.8)
      IF(T-TOL)1,1,3
    1 CONTINUE
      N=50
C     N=NUMBER POINTS DESIRED IN INITIAL EXPANSION FAN
      IPGO=0
      NQI=0
      NCSI=1
      NQE=N
      CALL T WAVE 1(NQE)
      JJ=2
      T=T+DT
      CALL T WAVE 2(NQE,NCS,N,JJ)
      NCS(1)=NCS
```

```
      IF(NPRNT-1)42,42,43
   42 JPRNT=0
   43 CONTINUE
      GO TO 32
    3 CONTINUE
      READ(60,205)IPGO,NQT,MCST,N
  205 FORMAT(I1,3I3)
      READ(60,206) (MCS(I),I=1,MCST)
      READ(60,206) (NQ(I),I=1,NQT)
  206 FORMAT(10I3)
      DO 7 I=1,NQT
    7 READ(60,207)GW(I)
  207 FORMAT(E15.8)
      DO 4 KQ=1,N
      READ(60,106)I,XG(KQ),PG(KQ),QG(KQ),SG(KQ)
  106 FORMAT(I3,4E15.8)
      UG(KQ)=.5*(PG(KQ)-QG(KQ))
      AG(KQ)=(G-1.)*(PG(KQ)+QG(KQ))/4.
    4 CONTINUE
      S2=SG(1)
      JPRNT=0
      IF(IPGO)8,8,9
    8 READ(60,107)PM1,PW1
  107 FORMAT(2F15.8)
    9 CONTINUE
      JPRNT=0
   32 CONTINUE
      DO 29 MT=1,NUMT
      T=T+DT
      JPRNT=JPRNT+1
      XXG(1)=XG(1)+DT*(UG(1)-AG(1))
      IF(XXG(1))500,500,510
  500 DO 501 I=1,10
  501 ZT(I)=2.*XG(I)/DT+UG(I)-AG(I)
      AAG(1)=AG(1)
  502 AGUESS=AAG(1)
      C1(1)=AAG(1)
      CALL INTER(2,10,1,ZT,XG,C1,XQ)
      CALL INTER(2,N,1,XG,CG,XQ,QQ)
      AAG(1)=(G-1.)*QQ(1)/2.
      IF(ABS(AAG(1)-AGUESS)-TOL)503,503,502
  503 XXG(1)=0.
      PPG(1)=2.*AAG(1)/(G-1.)
      QQG(1)=PPG(1)
      UUG(1)=0.
      SSG(1)=S2
      XGB(1)=0.

      XGQ(1)=XQ(1)
      DO 504 I=1,10
  504 ZT(I)=XG(I)+DT*(UG(I)-AG(I))/2.
      AAG(2)=AG(1)
      UUG(2)=UG(1)
  505 AGUESS=AAG(2)
      UGUESS=UUG(2)
      XXG(2)=XG(1)+DT*(UG(1)+AG(1)+UUG(2)+AAG(2))/2.
      C1(1)=XXG(2)-DT*(UUG(2)-AAG(2))/2.
      CALL INTER(2,10,1,ZT,XG,C1,XQ)
      CALL INTER(2,10,1,XG,QG,XQ,QQ)
      UUG(2)=(PG(1)-QQ(1))/2.
      AAG(2)=(G-1.)*(PG(1)+QQ(1))/4.
      IF(ABS(UUG(2)-UGUESS)-TOL)506,506,505
  506 IF(ABS(AAG(2)-AGUESS)-TOL)507,507,505
  507 PPG(2)=2.*AAG(2)/(G-1.)+UUG(2)
      QQG(2)=2.*AAG(2)/(G-1.)-UUG(2)
      SSG(2)=SG(1)
```

```
      XGR(2)=XG(1)
      XGQ(2)=XQ(1)
      DO 508 I=1,N
      IF(XG(I)-XQ(1))508,508,509
  508 CONTINUE
  509 KQ=I-1
      IQ=2
      GO TO 511
  510 CONTINUE
      PPG(1)=P2
      QQG(1)=Q2
      AAG(1)=A2
      UUG(1)=U2
      SSG(1)=S2
      XGR(1)=0.
      XGQ(1)=XG(1)
      IQ=1
      KQ=1
  511 CONTINUE
      DO 2 I=1,N
      Z1(I)=XG(I)+DT*(UG(I)+AG(I))/2.
    2 Z2(I)=XG(I)+UG(I)*DT/2.
      IIQ=1
      KKQ=1
      JQ=1
      JJQ=1
      JCS=1
      JJCS=1
  400 CONTINUE
      IQ=IQ+1
      NN=IQ
      KQ=KQ+1
      CALL POINTQ(IIQ,KKQ,T,IQ,KQ,N,0,DTP)
      DIX=ABS(XXG(IQ)-XXG(IQ-1))
      DIQ=ABS(QQG(IQ)-QQG(IQ-1))
      IF(XXG(IQ)-XXG(IQ-1))427,427,401
  401 IF(XXG(IQ)-XTL)402,406,406
  402 IF(DIX-DXOMAX)403,403,417
  403 IF(DIQ-DQOMAX)404,404,417
  404 IF(DIX-DXOMIN)405,405,422
  405 IF(DIQ-DQOMIN)418,418,422
  406 IF(XXG(IQ).GT.XTL.AND.XXG(IQ).LE.XTR)407,411
  407 IF(DIX-DXIMAX)408,408,417
  408 IF(DIQ-DQOMAX)409,409,417
  409 IF(DIX-DXIMIN)410,410,422
  410 IF(DIQ-DQIMIN)418,418,422
  411 IF(XXG(IQ)-1.)412,412,412
  412 IF(DIX-DXOMAX)416,416,413
  413 KKK=IQ-1
      CALL POINTADD(IQ,N)
      DO 414 KKJ=KKK,IQ
      IF(XXG(KKJ)-1.)414,415,415
  414 CONTINUE
  415 IQ=KKJ
      NN=IQ
  416 CALL SUPEND(IQ,KQ,N,NN,IPGO)
      GO TO 51
  417 CALL POINTADD(IQ,N)
      NN=IQ
      GO TO 422
  418 IF(KQ-NQ(JQ))419,423,419
  419 IF(KQ-MCS(JCS))420,425,420
  420 IF(KQ-N)421,23,421
  421 IQ=IQ-1
      GO TO 400
  422 IF(KQ-NQ(JQ))424,423,424
```

```
423   CALL QOCK(T,IQ,KQ,JQ,JJQ,JCS,JJCS,N,NN,IPGO)
424   IF(KQ-MCS(JCS))426,425,426
425   CALL CONT(T,IQ,KQ,JQ,JJQ,JCS,JJCS,N,IIQ,KKQ,NN,IPGO)
426   IF(KQ-N)400,23,400
427   CALL CROSS(IQ,KQ,JQ,JJQ,NEWQ)
      IF(NEWQ)422,422,428
428   JPRNT=NPRNT
      GO TO 422
   23 IF(IPGO)26,24,26
   24 CALL PHOCK(IQ,N,NN)
      IF(XXG(NN)-1.)51,51,83
   83 IF(UUG(IQ)-AAG(IQ))84,84,25
   84 NN=NN-1
      CALL SUBEND(NN,N)
      IPGO=1
      GO TO 51
   25 IQ=NN
      CALL SUPEND(IQ,KQ,N,NN,IPGO)
   26 CONTINUE
      IF(IPGO)51,51,49
   49 IF(XXG(NN)-1.)50,51,51
   50 CALL SUBEND(NN,N)
   51 CONTINUE
      NNQT=JJQ-1
      MMCST=JJCS-1
      IF(JPRNT-NPRNT)28,27,28
   27 CALL PRINTOUT(T,NN,IPGO,XTEST)
      JPRNT=0
   28 CONTINUE
      CALL MOVE(NN,N,IPGO)
      GO TO (429,29),SSWTCHF(4)
   29 CONTINUE
429   WRITE(61,308)T
  308 FORMAT(3X,5HTIME=,E15.3//)
      WRITE(61,101)
  101 FORMAT(12X,1HX,15X,1HP,14X,1HQ,14X,1HA,14X,1HU,14X,1HS,13X,4HMACH,
     110X,5HPRESS)
      DO 30 I=1,N
      AMACH=UG(I)/AG(I)
      PRESS=AG(I)**Q*EXP(-G*SG(I))
   30 WRITE(61,105)I,XG(I),PG(I),QG(I),AG(I),UG(I),SG(I),AMACH,PRESS
  105 FORMAT(3X,I3,8E15.8)
      WRITE(62,86)T
      WRITE(62,205)IPGO,NQT,MCST,N
      WRITE(62,206) (MCS(I),I=1,MCST)
      WRITE(62,206) (NQ(I),I=1,NQT)
      DO 34 I=1,NQT
   34 WRITE(62,207)QW(I)
      DO 35 I=1,N
   35 WRITE(62,106)I,XG(I),PG(I),QG(I),SG(I)
      IF(IPGO)37,36,37
   36 WRITE(62,107)PM1,PW1
   37 CONTINUE
      STOP
      END
```

```
      SUBROUTINE POINTADD(IQ,N)
      DIMENSION XB(1),AB(1),UB(1),SB(1),XC(1),UC 1),SC(1),XS(1),SS(1),C1
     1(1),C2(1),C3(1),Z3(300),AC(1)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XT .XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350) UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG( 50),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      DO 10 I=1,N
   10 Z3(I)=XG(I)+.5*(UG(I)-AG(I))*DT
      DXX=XXG(IQ)-XXG(IQ-1)
      DADX=(AAG(IQ)-AAG(IQ-1))/DXX
      DUDX=(UUG(IQ)-UUG(IQ-1))/DXX
      IF(XXG(IQ).GE.XTL.AND.XXG(IQ).LE.XTR)1,2
    1 TOLQC=.15
      GO TO 3
    2 TOLQC=TOLQ
    3 DQ=QQG(IQ)-QQG(IQ-1)
      DO 11 I=2,4
      AI=I
      DDQ=DQ/AI
      IF(ABS(DDQ).GT.TOLQC)11,12
   11 CONTINUE
      IH=3
      GO TO 13
   12 IH=I-1
   13 H=DXX/AI
      IA=IQ+IH
      XXG(IA)=XXG(IQ)
      PPG(IA)=PPG(IQ)
      QQG(IA)=QQG(IQ)
      AAG(IA)=AAG(IQ)
      UUG(IA)=UUG(IQ)
      SSG(IA)=SSG(IQ)
      XGB(IA)=XGB(IQ)
      XGQ(IA)=XGQ(IQ)
      IQ=IQ-1
      DO 19 J=1,IH
      IQ=IQ+1
      XA=XXG(IQ-1)+H
      CALL AREA(XA,DAA)
      JC=0
      AA=AAG(IQ-1)+DADX*(XA-XXG(IQ-1))
      UA=UUG(IQ-1)+DUDX*(XA-XXG(IQ-1))
   14 AGUESS=AA
      UGUESS=UA
      JC=JC+1
      C1(1)=XA-.5*(UA+AA)*DT
      C3(1)=XA-.5*(UA-AA)*DT
      C2(1)=XA-.5*UA*DT
      CALL INTER(2,N,1,Z1,XG,C1,XB)
      IF(XB(1)-XG(1))20,20,21
   20 AB(1)=AG(1)
      UB(1)=UG(1)
      SB(1)=SG(1)
      GO TO 22
   21 CONTINUE
      CALL INTER(2,N,1,XG,AG,XB,AB)
      CALL INTER(2,N,1,XG,UG,XB,UB)
      CALL INTER(2,N,1,XG,SG,XB,SB)
```

```
 22 CONTINUE
    CALL INTER(2,N,1,Z2,XG,C2,XS)
    IF(XS(1)-XG(1))23,23,24
 23 SS(1)=SG(1)
    GO TO 25
 24 CONTINUE
    CALL INTER(2,N,1,XG,SG,XS,SS)
 25 CONTINUE
    CALL INTER(2,N,1,Z3,XG,C3,XC)
    CALL INTER(2,N,1,XG,AG,XC,AC)
    CALL INTER(2,N,1,XG,UG,XC,UC)
    CALL INTER(2,N,1,XG,SG,XC,SC)
    SA=SS(1)
    CALL AREA(XB(1),DAB)
    CALL AREA(XC(1),DAC)
    PB=2.*AB(1)/(G-1.)+UB(1)
    QC=2.*AC(1)/(G-1.)-UC(1)
    PA=PB+(-UA*AA*DAA-UB(1)*AB(1)*DAB)*DT/2.+( A+AB(1))*(SA-SB(1))/2.
    QA=QC+(-UA*AA*DAA-UC(1)*AC(1)*DAC)*DT/2.+( A+AC(1))*(SA-SC(1))/2.
    AA=(G-1.)*(PA+QA)/4.
    UA=.5*(PA-QA)
    IF(JC-25)16,15,15
 15 WRITE(61,102)
102 FORMAT(/3X,35HTOLERANCE CANNOT BE MET IN P INTADD)
    WRITE(61,103)UGUESS,UA,AGUESS,AA
103 FORMAT(3X,7HUGUESS=,E15.8,2X,3HUA=,E15.8,2 ,7HAGUESS=,E15.8,2X,3HA
   1A=,E15.8/)
    GO TO 18
 16 CONTINUE
    IF(ABS(UA-UGUESS)-TOL)17,17,14
 17 IF(ABS(AA-AGUESS)-TOL)18,18,14
 18 XXG(IQ)=XA
    PPG(IQ)=2.*AA/(G-1.)+UA
    QQG(IQ)=2.*AA/(G-1.)-UA
    AAG(IQ)=AA
    UUG(IQ)=UA
    SSG(IQ)=SA
    XGB(IQ)=XB(1)
    XGQ(IQ)=XC(1)
 19 CONTINUE
    IQ=IA
    RETURN
    END
```

```
      SUBROUTINE POINTQ(IIQ,KKQ,T,IQ,KQ,N,ID,DTP)
      DIMENSION C1(1),XB(2),AB(1),UB(1),SB(1),C2(1),XS(2),SS(1),DTT(2)
      DIMENSION USS(1),PBB(1)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      JC=0
      CALL AREA(XG(KQ),DAQ)
      AA=AG(KQ)
      UA=UG(KQ)
   19 AGUESS=AA
      UGUESS=UA
      JC=JC+1
      XA=XG(KQ)+DT*(UA-AA+UG(KQ)-AG(KQ))/2.
      C1(1)=XA-.5*(UA+AA)*DT
      CALL INTER(2,N,1,Z1,XG,C1,XB)
      IF(ID)23,20,23
   20 IF(XB(1)-XG(1))21,21,22
   21 AB(1)=A2
      UB(1)=U2
      SB(1)=S2
      GO TO 28
   22 DTP=DT
      CALL INTER(2,N,1,XG,AG,XB,AB)
      CALL INTER(2,N,1,XG,PG,XB,PBB)
      UB(1)=PBB(1)-2.*AB(1)/(G-1.)
      CALL INTER(2,N,1,XG,SG,XB,SB)
      GO TO 28
   23 DXX=XXG(IIQ)-XG(KKQ)
      IF(XB(1)-XG(KKQ))24,24,22
   24 DTT(1)=(XA-XXG(IIQ))/(UA+AA-DXX/DT)
      IF(XA-XXG(IIQ))1,1,25
    1 SA=SG(KQ)
      XB(1)=0.
      GO TO 41
   25 UB(1)=UUG(IIQ)-DTT(1)*(UUG(IIQ)-UG(KKQ))/DT
      AB(1)=AAG(IIQ)-DTT(1)*(AAG(IIQ)-AG(KKQ))/DT
      DTT(2)=(XA-XXG(IIQ))/((UA+AA+UB(1)+AB(1))/2.-DXX/DT)
      IF(DTT(2))59,05,58
   58 IF(DTT(2)-DT)05,65,59
   59 XB(1)=XA-DT*(UA+AA)
   60 CALL INTER(2,N,1,XG,AG,XB,AB)
      CALL INTER(2,N,1,XG,UG,XB,UB)
      XB(2)=XA-DT*(UA+AA+UB(1)+AB(1))/2.
      IF(XB(2)-XG(KKQ))61,61,62
   61 XB(1)=XG(KKQ)
      AB(1)=AG(KKQ)
      UB(1)=UG(KKQ)
      SB(1)=SG(KKQ)
      DTP=DT
      GO TO 28
   62 CONTINUE
      IF(ABS(XB(2)-XB(1))-TOL)64,64,63
   63 XB(1)=XB(2)
```

```
      GO TO 60
   64 DTP=DT
      CALL INTER(2,N,1,XG,SG,XB,SB)
      GO TO 28
   65 CONTINUE
      IF(ABS(DTT(2)-DTT(1))=.1E=07)27,27,26
   26 DTT(1)=DTT(2)
      GO TO 25
   27 SB(1)=SSG(IIQ)-DTT(1)*(SSG(IIQ)-SG(KKQ))/DT
      XB(1)=XXG(IIQ)-DTT(1)*DXX/DT
      DTP=DTT(1)
   28 PB=2.*AB(1)/(G-1.)+UE(1)
      C2(1)=XA-.5*DT*UA
      CALL INTER(2,N,1,Z2,XG,C2,XS)
      IF(ID)29,29,32
   29 IF(XS(1)-XG(1))30,30,31
   30 SS(1)=S2
      GO TO 37
   31 CALL INTER(2,N,1,XG,SG,XS,SS)
      GO TO 37
   32 IF(XS(1)-XG(KKQ))33,33,29
   33 DTT(1)=(XA-XXG(IIQ))/(UA-DXX/DT)
   34 US=UUG(IIQ)-DTT(1)*(UUG(IIQ)-UG(KKQ))/DT
      DTT(2)=(XA-XXG(IIQ))/((UA+US)/2.-DXX/DT)
      IF(DTT(2))51,21,50
   50 IF(DTT(2)-DT)57,57,51
   51 XS(1)=XA-UA*DT
   52 CALL INTER(2,N,1,XG,AG,XS,USS)
      XS(2)=XA-DT*(UA+USS(1))/2.
      IF(XS(2)-XG(KKQ))53,53,54
   53 SS(1)=SG(KKQ)
      GO TO 37
   54 IF(ABS(XS(2)-XS(1))-TOL)56,56,55
   55 XS(1)=XS(2)
      GO TO 52
   56 CALL INTER(2,N,1,XG,SG,XS,SS)
      GO TO 37
   57 CONTINUE
      IF(ABS(DTT(2)-DTT(1))=.1E=07)36,36,35
   35 DTT(1)=DTT(2)
      GO TO 34
   36 SS(1)=SSG(IIQ)-DTT(1)*(SSG(IIQ)-SG(KKQ))/DT
   37 SA=SS(1)
      CALL AREA(XA,DAA)
      CALL AREA(XB(1),DAB)
      PA=PB+(-UA*AA*DAA-UB(1)*AB(1)*DAB)*DTP/2.+(AA+AB(1))*(SA-SB(1))/2.
      QA=QG(KQ)+(-UG(KQ)*AG(KQ)*DAQ-UA*AA*DAA)*DT/2.+(AA+AG(KQ))*(SA-SG(
     1KQ))/2.
      AA=(G-1.)*(PA+QA)/4.
      UA=.5*(PA-QA)
      IF(JC-25)39,38,38
   38 WRITE(61,100)
  100 FORMAT(/3X,33HTOLERANCE CANNOT BE MET IN POINTQ)
      UDIFF=UGUESS-UA
      ADIFF=AGUESS-AA
      WRITE(61,101)XA,UDIFF,ADIFF
  101 FORMAT(3X,2HX=,E15.8,2X,6HUDIFF=,E15.8,2X,6HADIFF=,E15.8/)
      GO TO 41
   39 IF(ABS(UA-UGUESS)-TOL)40,40,19
   40 IF(ABS(AA-AGUESS)-TOL)41,41,19

   41 XXG(IQ)=XA
      UUG(IQ)=UA
      AAG(IQ)=AA
      PPG(IQ)=2.*AA/(G-1.)+UA
      QQG(IQ)=2.*AA/(G-1.)-UA
      SSG(IQ)=SA
      XGB(IQ)=XB(1)
      XGQ(IQ)=XG(KQ)
      RETURN
      END
```

```
      SUBROUTINE T WAVE 1(NQE)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL.XTR,XD
      COMMON XXG(350).PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      P1P1=1.
      C=2.*G/(G-1.)
      S2=S1-ALOG(P2P1/A2**C)/G
      P1=2.*A1/(G-1.)+U1
      Q1=2.*A1/(G-1.)-U1
      P2=2.*A2/(G-1.)+U2
      Q2=2.*A2/(G-1.)-U2
      S4=S2
      P4=P2
C     ITERATION PROCEDURE BEGINS
      U3=1.2
    1 U3G=U3
      A=-(G+1.)*(U1-U3)/A1
      Z=(A+SQRT(A**2+16.))/4.
      A3A1=SQRT((2.+(G-1.)*Z**2)*(2.*G*Z**2-(G-1.)))/((G+1.)*Z)
      AG1=2.*G*Z**2/(G+1.)-(G-1.)/(G+1.)
      AG2=((1.+(G-1.)*Z**2/2.)/((G+1.)*Z**2/2.))**G
      DS=ALOG(AG1*AG2)/(G*(G-1.))
      P3P1=A3A1**(2.*G/(G-1.))*EXP(-G*DS)
      P4P1=P3P1
      A4=(P4P1*EXP(G*(S4-S1)))**((G-1.)/(2.*G))*A1
      U4=P4-2.*A4/(G-1.)
      U3=U4
      IF(ABS(U3-U3G)-TOL)2,2,1
    2 CONTINUE
C     ITERATION COMPLETED
      PM1=7
      Q4=2.*A4/(G-1.)-U4
      P4=2.*A4/(G-1.)+U4
      A3=A3A1*A1
      Q3=2.*A3/(G-1.)-U3
      P3=2.*A3/(G-1.)+U3
      S3=S1+DS
      PW1=U1+A1*PM1
      T=0.
      WRITE(61,115)
  115 FORMAT(//3X,5HTIME=,E15.8//)
      WRITE(61,117)
  117 FORMAT(5X,1HI,7X,1HX,14X,1HP,14X,1HQ,14X,1HA,14X,1HU,14X,1HS/)
      AQ=NQE-1
      DQ=(Q2-Q4)/AQ
      DO 3 KQ=1,NQE
      QI=KQ-1
      XG(KQ)=XD
      PG(KQ)=P2
      QG(KQ)=Q2-DQ*QI
      UG(KQ)=.5*(PG(KQ)-QG(KQ))
      AG(KQ)=(G-1.)*(PG(KQ)+QG(KQ))/4.
      SG(KQ)=S2
      WRITE(61,101)KQ,XG(KQ),PG(KQ),QG(KQ),AG(KQ),UG(KQ),SG(KQ)
  101 FORMAT(3X,I3,6E15.8)
    3 CONTINUE
```

```
      WRITE(61,131)
  131 FORMAT(/3X,42HPROPERTIES ON LEFT SIDE OF CONTACT SURFACE)
      WRITE(61,132)P4,Q4,U4,A4,S4
  132 FORMAT(3X,2HP=,E15.8,2HQ=,E15.8,2HU=,E15.8,2HA=,E15.8,2HS=,E15.8/)
      WRITE(61,133)
  133 FORMAT(/3X,43HPROPERTIES ON RIGHT SIDE OF CONTACT SURFACE)
      WRITE(61,132)P3,Q3,U3,A3,S3
      WRITE(61,134)
  134 FORMAT(/3X,37HPROPERTIES ON LEFT SIDE OF SHOCK WAVE)
      WRITE(61,132)P3,Q3,U3,A3,S3
      WRITE(61,135)PM1,PW1
  135 FORMAT(/3X,20HP SHOCK MACH NUMBER=,E15.8,/3X,17HP SHOCK VELOCITY=,
     1E15.8)
      WRITE(61,136)
  136 FORMAT(/3X,38HPROPERTIES ON RIGHT SIDE OF SHOCK WAVE)
      WRITE(61,132)P1,Q1,U1,A1,S1
      WRITE(61,110)
  110 FORMAT(1H1)
      RETURN
      END
```

```
      SUBROUTINE T WAVE 2(NQE,NCS,N,JJ)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      TC=JJ-1
      T=TC*DT
      XXG(1)=XD+DT*(U2-A2)
      PPG(1)=P2
      QQG(1)=Q2
      AAG(1)=A2
      UUG(1)=U2
      SSG(1)=S2
      DO 30 I=2,NQE
      XXG(I)=XD+DT*(UG(I)-AG(I))
      PPG(I)=PG(I)
      QQG(I)=QG(I)
      AAG(I)=AG(I)
      UUG(I)=UG(I)
   30 SSG(I)=SG(I)
      XCS=XD+U3*DT
      NQ2=2
      AQ2=NQ2
      DX2=(XCS-XXG(NQE))/AQ2
      K1=NQE+1
      NCS=NQE+NQ2
      DO 31 K=K1,NCS
      XXG(K)=XXG(K-1)+DX2
      PPG(K)=PPG(NQE)
      QQG(K)=QQG(NQE)
      AAG(K)=AAG(NQE)
      UUG(K)=UUG(NQE)
      SSG(K)=SSG(NQE)
   31 CONTINUE
      XSP=XD+DT*PW1
      NQ3=10
      AQ3=NQ3-1
      DX3=(XSP-XXG(NCS))/AQ3
      K2=NCS+1
      N=NCS+NQ3
      DO 32 K=K2,N
      AK=K-K2
      XXG(K)=XXG(NCS)+DX3*AK
      PPG(K)=P3
      QQG(K)=Q3
      AAG(K)=A3
      UUG(K)=U3
   32 SSG(K)=S3
      PPM1=PM1
      PPW1=PW1
      WRITE(61,115)T
  115 FORMAT(//3X,5HTIME=,E15.8,1X,3H(*)//)
      WRITE(61,117)
  117 FORMAT(5X,1HI,7X,1HX,14X,1HP,14X,1HQ,14X,1HA,14X,1HU,14X,1HS/)
      DO 33 I=1,NCS
   33 WRITE(61,101)I,XXG(I),PPG(I),QQG(I),AAG(I),UUG(I),SSG(I)
  101 FORMAT(3X,I3,6E15.8)
```

```
      WRITE(61,137)
  137 FORMAT(3X,15HCONTACT SURFACE)
      NCSR=NCS+1
      DO 34 I=NCSR,N
   34 WRITE(61,101)I,XXG(I),PPG(I),QQG(I),AAG(I),UUG(I),SSG(I)
      WRITE(61,135)PPM1,PPW1
  135 FORMAT(/3X,20HP SHOCK MACH NUMBER=,E15.8,/3X,17HP SHCCK VELOCITY=,
     1E15.8)
      WRITE(61,110)
  110 FORMAT(1H1)
      DO 35 I=1,N
      XG(I)=XXG(I)
      PG(I)=PPG(I)
      QG(I)=QQG(I)
      AG(I)=AAG(I)
      UG(I)=UUG(I)
   35 SG(I)=SSG(I)
      PM1=PPM1
      PW1=PPW1
      RETURN
      END
```

```
      SUBROUTINE CONT(T,IQ,KQ,JQ,JJQ,JCS,JJCS,N,IIQ,KKQ,NN,IPGO)
      COMMON G,DT,TOL,TOLQ
      COMMON XF(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),LCONTACT(10)
    1 CALL CONTACT(IQ,KQ,JQ,JJQ,JCS,JJCS,N,IIQ,KKQ,NN,IPGO,T)
      K=IQ-1
      IF(KQ-N)18,13,13
   18 IF(KQ-MCS(JCS))2,1,2
    2 IQ=IQ+1
      KQ=KQ+1
      NN=IQ
      CALL POINTQ(IIQ,KKQ,T,IQ,KQ,N,1,DTP)
      IF(KQ-MCS(JCS))19,1,19
   19 IF(XXG(IQ)-1.)4,3,3
    3 CALL SUPEND(IQ,KQ,N,NN,IPGO)
      GO TO 13
    4 IF(KQ-NQ(JQ))8,5,8
    5 IF(XXG(IQ)-XXG(IQ-1))20,20,21
   20 IQ=IQ-1
   21 CALL QSHOCK(IQ,KQ,JQ,JJQ,JCS,JJCS,N,IIQ,KKQ)
      IF(XSHOCK(JJQ-1)-XCONTACT(JJCS-1))6,6,7
    6 CALL CSCROSSQ(T,IQ,KQ,JQ,JJQ,JCS,JJCS,IIQ,KKQ,N)
    7 GO TO 2
    8 IF(KQ-N)9,13,13
    9 IF(DTP)11,11,10
   10 IF(XXG(IQ)-XXG(IQ-1))11,11,12
   11 IQ=IQ-1
      GO TO 2
   12 IF(ABS(DTP-DT)-TOL)13,13,2
   13 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CONTACT(IC,KQ,JQ,JJQ,JCS,JJCS,N,IIQ,KKQ,NN,IPGO,T)
      DIMENSION XP(2),AP(1),UP(1),SP(1),C1(1),XB(2),UB(1),AB(1),SB(1),XC
     1(2),UC(1),AC(1),C2(1),XS(1),SS(1),SC(1)
      DIMENSION XEND(1),AEND(1),UEND(1),SEND(1)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),LCONTACT(10)
      NCS=MCS(JCS)
      NCS2=MCS(JCS+1)
      CALL QDIS(JQ,XDISL,XDISR)
      SL=SG(NCS)
      SR=SG(NCS+1)
      O1=EXP((G-1.)*(SL-SR)/2.)
      E=1.+EXP((G-1.)*(SR-SL)/2.)
      DTQ=DT
      UL=UG(NCS)
      AL=AG(NCS)
      JC=0
    1 U=UL
      A=AL
      JC=JC+1
      IZ=0
      IP=0
      UR=UL
      AR=AL/O1
      XR=XG(NCS)+DT*(UR+UG(NCS+1))/2.
      XL=XR
      XP(1)=XR-DT*(UR-AR)
      IF(XR-1.)2,32,32
    2 CONTINUE
      IF(XP(1)-1.)31,32,32
   31 CONTINUE
      CALL INTER(2,N,1,XG,AG,XP,AP)
      CALL INTER(2,N,1,XG,UG,XP,UP)
      XP(2)=XR-DT*(UR-AR+UP(1)-AP(1))/2.
      IF(JCS-MCST)35,36,36
   35 IF(XP(2)-XG(NCS2))36,82,82
   36 CONTINUE
      IF(ABS(XP(2)-XP(1))-TOL)4,4,3
    3 XP(1)=XP(2)
      GO TO 2
   32 CONTINUE
      XEND(1)=1.
      CALL INTER(2,IQ,1,XXG,AAG,XEND,AEND)
      CALL INTER(2,IQ,1,XXG,UUG,XEND,UEND)
      CALL INTER(2,IQ,1,XXG,SSG,XEND,SEND)
      NN=IQ+1
      KQ=N
      IPGO=1
      XXG(NN)=1.000001
      AAG(NN)=AEND(1)
      UUG(NN)=UEND(1) .
```

```
      SSG(NN)=SEND(1)

      PPG(NN)=2.*AAG(NN)/(G=1.)+UUG(NN)
      QQG(NN)=2.*AAG(NN)/(G=1.)-UUG(NN)
      XGR(NN)=0.
      XGQ(NN)=0.
      GO TO 75
    4 IF(JCS-MCST)81,83,83
   81 NCS2=MCS(JCS+1)
      IF(XP(1)-XG(NCS2))83,82,82
   82 UP(1)=UG(NCS2)
      AP(1)=AG(NCS2)
      SP(1)=SG(NCS2)
      SM1=UG(NCS2)
      SM2=(UR-AR+UP(1)-AP(1))/2.
      TQ=T+(XR-XG(NCS2)-DT*SM1)/(SM1-SM2)
      XQ=XR+SM2*(TQ-T)
      DTQ=T-TQ
      XP(1)=XQ
      IP=1
      GO TO 6
   83 CALL INTER(2,N,1,XG,SG,XP,SP)
      IF(XP(1)-XDISR)6,5,5
    5 K1=NQ(JQ)
      IZ=1
      XP(1)=XG(K1)
      AP(1)=AG(K1)
      UP(1)=UG(K1)
      SP(1)=SG(K1)
    6 QP=2.*AP(1)/(G-1.)-UP(1)
      CALL AREA(XR,DAR)
      CALL AREA(XP(1),DAP)
      QR=QP+(-UP(1)*AP(1)*DAP-UR*AR*DAR)*DTQ/2.+(AP(1)+AR)*(SR-SP(1))/2.
      C1(1)=XL-DT*(UL+AL)/2.
      CALL INTER(2,N,1,Z1,XG,C1,XB)
      IF(XB(1)-XDISL)60,60,64
   60 JL=NNQ(JJQ-1)+1
      JI=NQ(JQ-1)+1
      DUB=(UUG(JL)-UG(JI))/DT
      DAB=(AAG(JL)-AG(JI))/DT
      DXQ=(QQW(JJG-1)+QW(JG=1))/2.
      XB(1)=XL+(UL+AL)*(XXG(JL)-XL)/(UL+AL-DXQ)
   61 TBT=(XB(1)-XXG(JL))/DXQ
      UB(1)=UUG(JL)+DUB*TBT
      AB(1)=AAG(JL)+DAB*TBT
      DXPW=(UL+AL+UB(1)+AB(1))/2.
      XB(2)=XL+DXPW*(XXG(JL)-XL)/(DXPW=DXQ)
      IF(ABS(XB(2)-XB(1))-TOL)63,63,62
   62 XB(1)=XB(2)
      GO TO 61
   63 SB(1)=SSG(JL)+(SSG(JL)-SG(JI))*TBT/DT
      DTP=-TBT
      GO TO 65
   64 CONTINUE
      IF(JJCS-2)89,84,84
   84 NCS1=MCS(JCS-1)+1
      IF(XB(1)-XG(NCS1))85,85,89
   85 NCCS1=MMCS(JJCS-1)+1
      DXX=XCONTACT(JJCS-1)-XG(NCS1)
      DTT1=(XL-XCONTACT(JJCS-1))/(UL+AL-DXX/DT)
   86 UB(1)=UCONTACT(JJCS-1)-DTT1*(UCONTACT(JJCS-1)-UG(NCS1))/DT
```

```
      AB(1)=AAG(NCCS1)-DTT1*(AAG(NCCS1)-AG(NCS1))/DT
      DTT2=(XL-XCONTACT(JJCS-1))/((UL+AL+UB(1)+AB(1))/2.-DXX/DT)
      IF(ABS(DTT2-DTT1)-.1E-07)88,88,87
   87 DTT1=DTT2
      GO TO 85
   88 SB(1)=SG(NCS1)
      XB(1)=XCONTACT(JJCS-1)-DTT1*DXX/DT
      DTP=DTT1
      GO TO 65
   85 CONTINUE
      CALL INTER(2,N,1,XG,UG,XB,UB)
      CALL INTER(2,N,1,XG,AG,XB,AB)
      CALL INTER(2,N,1,XG,SG,XB,SB)
      DTP=DT
   65 PB=2.*AB(1)/(G-1.)+UB(1)
      CALL AREA(XB(1),DAB)
      PL=PB+(-UB(1)*AB(1)*DAB-UL*AL*DAR)*DTP/2.+(AL+AB(1))*(SL-SB(1))/2.
      AL=(G-1.)*(PL+QR)/(2.*E)
      UL=PL-2.*AL/(G-1.)
      IF(JC-30)21,20,20
   20 UDIFF=U-UL
      ADIFF=A-AL
      WRITE(61,90)
   90 FORMAT(/3X,28HTOLERANCE NOT MET IN CONTACT)
      WRITE(61,91)XL,ADIFF,UDIFF
   91 FORMAT(3X,3HXL=,E15.8,2X,6HADIFF=,E15.8,2X,6HUDIFF=,E15.8)
      GO TO 11
   21 CONTINUE
      IF(ABS(UL-U)-TOL)9,9,10
    9 IF(ABS(AL-A)-TOL)11,11,10
   10 GO TO 1
   11 CONTINUE
      NCSL=IQ+1
      NCSR=NCSL+1
      MMCS(JJCS)=NCSL
      XCONTACT(JJCS)=XR
      UCONTACT(JJCS)=UR
      XXG(NCSL)=XL
      PPG(NCSL)=PL
      AAG(NCSL)=AL
      UUG(NCSL)=UL
      QQG(NCSL)=2.*AL/(G-1.)-UL
      SSG(NCSL)=SL
      XGR(NCSL)=XB(1)
      XGQ(NCSL)=0.
      XXG(NCSR)=XR
      QQG(NCSR)=QR
      AAG(NCSR)=AR
      UUG(NCSR)=UR
      PPG(NCSR)=2.*AR/(G-1.)+UR
      SSG(NCSR)=SR
      XGR(NCSR)=0.
      XGQ(NCSR)=XP(1)
      IQ=NCSR
      NN=IQ
      IIQ=IQ
      IF(IZ)51,51,50
   50 KQ=K1-1
      KKQ=KQ
      GO TO 52
   51 IF(IP)54,54,53
   53 KQ=NCS2


      GO TO 52

   54 DO 66 I=1,N
      IF(XG(I)-XP(1))66,66,67
   66 CONTINUE
      KQ=N
      KKQ=KQ
      GO TO 52
   67 KQ=I-1
      KKQ=NCS+1
   52 JCS=JCS+1
      JJCS=JJCS+1
   75 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GOWK(T,IQ,KQ,JQ,JJQ,JCS,JJCS,N,NN,IPGO)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),LCONTACT(10)
   17 CALL QSHOCK(IQ,KQ,JQ,JJQ,JCS,JJCS,N,IIQ,KKQ)
      NN=IQ
      IF(JJQ-2)20,20,18
   18 IF(XSHOCK(JJQ-1)-XSHOCK(JJQ-2))19,19,20
   19 CALL QCROSSQ(I,IQ,KQ,JQ,JJQ,JCS,JJCS,IIQ,KKQ,N)
   20 CONTINUE
   21 IQ=IQ+1
      KQ=KQ+1
      NN=IQ
      CALL POINTQ(IIQ,KKQ,T,IQ,KQ,N,1,DTP)
      IF(XXG(IQ)-1.)23,22,22
   22 CALL SUPEND(IQ,KQ,N,NN,IPGO)
      GO TO 28
   23 IF(KQ-NQ(JQ))25,24,25
   24 GO TO 17
   25 IF(KQ-MCS(JCS))26,10,26
   10 IF(XXG(IQ)-XXG(IQ-1))11,11,28
   11 IQ=IQ-1
      GO TO 28
   26 IF(KQ-N)27,28,28
   27 CONTINUE
      IF(DTP)15,15,14
   14 IF(XXG(IQ)-XXG(IQ-1))15,15,16
   15 IQ=IQ-1
      GO TO 21
   16 CONTINUE
      IF(ABS(DTP-DT)-TOL)28,28,21
   28 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GSHOCK(IQ,KQ,JQ,JJQ,JCS,JJCS,N,IIQ,KKQ)
      DIMENSION XL(1),AL(1),UL(1),SL(1),XC(2),AC(1),UC(1),SC(1)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,FPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),LCONTACT(10)
      NL=NQ(JQ)
      CALL XDISCON(JQ,JCS,XDISR,NQOCS)
      JC=0
      QQSW=QW(JQ)
   10 GUESS=QQSW
      IZ=0
      JC=JC+1
      WA=(QQSW+QW(JW))/2.
      XL(1)=XG(NL)+DT*WA
      XR=XL(1)
      IF(JJCS-1)2,2,1
    1 IF(XCONTACT(JJCS-1)-XL(1))2,4,4
    2 IF(JJQ-1)5,5,3
    3 IF(XSHOCK(JJQ-1)-XL(1))5,4,4
    4 AL(1)=AG(NL)
      UL(1)=UG(NL)
      SL(1)=SG(NL)
      GO TO 6
    5 CONTINUE
      CALL INTER(2,IQ,1,XXG,AAG,XL,AL)
      CALL INTER(2,IQ,1,XXG,UUG,XL,UL)
      CALL INTER(2,IQ,1,XXG,SSG,XL,SL)
    6 CONTINUE
      PL=2.*AL(1)/(G-1.)+UL(1)
      QL=2.*AL(1)/(G-1.)-UL(1)
      QQSM=-(QQSW-UL(1))/AL(1)
      DU=2.*(1.-QQSM**2)/((G+1.)*QQSM)
      UR=UL(1)+DU*AL(1)
      AR=(SQRT((2.+(G-1.)*QQSM**2)*(2.*G*QQSM**2-(G-1.)))/((G+1.)*QQSM))
     1*AL(1)
      XC(1)=XR-DT*(UR-AR)
   11 CONTINUE
      CALL INTER(2,N,1,XG,AG,XC,AC)
      CALL INTER(2,N,1,XG,UG,XC,UC)
      XC(2)=XR-DT*(UR-AR+UC(1)-AC(1))/2.
      IF(ABS(XC(2)-XC(1))-TOL)13,13,12
   12 XC(1)=XC(2)
      GO TO 11
   13 CALL INTER(2,N,1,XG,SG,XC,SC)
      IF(XC(1)-XDISR)17,17,14
   14 IF(NQOCS-1)17,15,16
   15 K1=NQ(JQ+1)
      IZ=1
      XC(1)=XG(K1)
      AC(1)=AG(K1)
      UC(1)=UG(K1)
      SC(1)=SG(K1)
```

```
      GO TO 17
   16 K1=MCS(JCS)
      IZ=1
      XC(1)=XG(K1)
      AC(1)=AG(K1)
      UC(1)=UG(K1)
      SC(1)=SG(K1)
   17 CALL AREA(XC(1),DAC)
      CALL AREA(XR,DAR)
      QC=2.*AC(1)/(G-1.)-UC(1)
      QR=2.*AR/(G-1.)-UR
      DEL=(QR-QL)/AL(1)
      CALL SHOCK(DEL,QQSM,LRAT,ARAT,PRAT,DS)
      SR=SL(1)+DS
      QR=QC+(-AC(1)*UC(1)*DAC-AR*UR*DAR)*DT/2.+(AR+AC(1))*(SR-SC(1))/2.
      DEL=(QR-QL)/AL(1)
      CALL SHOCK(DEL,QQSM,LRAT,ARAT,PRAT,DS)
      QQSW=UL(1)+AL(1)*QQSM
      IF(JC-40)19,19,18
   18 DIFF=ABS(QQSW-GUESS)
      WRITE(61,90)
   90 FORMAT(/3X,33HTOLERANCE CANNOT BE MET IN QSHOCK)
      WRITE(61,91)DIFF
   91 FORMAT(3X,14HTOLERANCE MET=,E15.8/)
      GO TO 20
   19 CONTINUE
      IF(ABS(QQSW-GUESS)-TOL)20,20,10
   20 QQW(JJQ)=QQSW
      QQM(JJQ)=QQSM
      DO 21 I=1,IG
      IF(XXG(I)-XR)21,22,22
   21 CONTINUE
   22 IQ=I
      NNQ(JJQ)=IQ
      XSHOCK(JJQ)=XL(1)
      XXG(IQ)=XL(1)
      PPG(IQ)=PL
      QQG(IQ)=QL
      AAG(IQ)=AL(1)
      UUG(IQ)=UL(1)
      SSG(IQ)=SL(1)
      XGR(IQ)=0.
      XGQ(IQ)=0.
      IQ=IQ+1
      IIQ=IQ
      XXG(IQ)=XR
      QQG(IQ)=QR
      AAG(IQ)=AAG(IQ-1)*ARAT
      UUG(IQ)=2.*AAG(IQ)/(G-1.)-QQG(IQ)
      PPG(IQ)=2.*AAG(IQ)/(G-1.)+UUG(IQ)
      SSG(IQ)=SSG(IQ-1)+DS
      XGR(IQ)=0.
      XGQ(IQ)=XC(1)
      IF(IZ)28,28,27
   27 KQ=K1-1
      KKQ=KQ
      GO TO 25
   28 CONTINUE
      DO 23 I=1,N
      IF(XG(I)-XC(1))23,23,24
   23 CONTINUE
      KQ=N-1
      KKQ=KQ
      GO TO 25
   24 KQ=I-1
      KKQ=NQ(JQ)+1
   25 JJQ=JJQ+1
      JQ=JQ+1
      RETURN
      END
```

```
      SUBROUTINE SUPEND(IQ,KQ,N,NN,IPGO)
      DIMENSION XEND(1),PEND(1),QEND(1),SEND(1)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      XEND(1)=1.
      CALL INTER(2,IQ,1,XXG,PPG,XEND,PEND)
      CALL INTER(2,IQ,1,XXG,QQG,XEND,QEND)
      CALL INTER(2,IQ,1,XXG,SSG,XEND,SEND)
      NN=IQ
      XXG(NN)=XEND(1)
      PPG(NN)=PEND(1)
      QQG(NN)=QEND(1)
      AAG(NN)=(G-1.)*(PPG(NN)+QQG(NN))/4.
      UUG(NN)=.5*(PPG(NN)-QQG(NN))
      SSG(NN)=SEND(1)
      XGB(NN)=0.
      XGQ(NN)=0.
      KQ=N
      IPGO=1
      RETURN
      END
```

```
      SUBROUTINE SUBEND(NN,N)
      COMMON G,DT,TOL,TOLQ
      COMMON XF(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      DIMENSION XB(1),AB(1),UB(1),SB(1),SIB(2)
      JCH=0
      NN=NN+1
      AM2=(XXG(NN-1)-XG(N))/DT
      XXG(NN)=1.
      CALL AREA(XXG(NN),DAE)
      UUG(NN)=UUG(NN-1)
   10 UGUESS=UUG(NN)
   20 AM1=UUG(NN)
      SIB(1)=(1.-AM1*DT-XG(N))/(AM2-AM1)
   11 UB(1)=UG(N)+SIB(1)*(UUG(NN-1)-UG(N))/DT
      AM1=(UUG(NN)+UB(1))/2.
      SIB(2)=(1.-AM1*DT-XG(N))/(AM2-AM1)
      IF(ABS(SIB(2)-SIB(1))-TOL)30,30,12
   12 SIB(1)=SIB(2)
      GO TO 11
   30 SSG(NN)=SG(N)+SIB(2)*(SSG(NN-1)-SG(N))/DT
      IF(JCH-1)32,31,31
   31 AAG(NN)=UUG(NN)
      GO TO 34
   32 AAG(NN)=EXP((G-1.)*SSG(NN)/2.)
   34 AM1=UUG(NN)+AAG(NN)
      SIB(1)=(1.-AM1*DT-XG(N))/(AM2-AM1)
   13 UB(1)=UG(N)+SIB(1)*(UUG(NN-1)-UG(N))/DT
      AB(1)=AG(N)+SIB(1)*(AAG(NN-1)-AG(N))/DT
      AM1=(UUG(NN)+AAG(NN)+UB(1)+AB(1))/2.
      SIB(2)=(1.-AM1*DT-XG(N))/(AM2-AM1)
      IF(ABS(SIB(2)-SIB(1))-TOL)15,15,14
   14 SIB(1)=SIB(2)
      GO TO 13
   15 SB(1)=SG(N)+SIB(2)*(SSG(NN-1)-SG(N))/DT
      XB(1)=XG(N)+AM2*SIB(2)
      DTP=DT-SIB(1)
      PB=2.*AB(1)/(G-1.)+UB(1)
      CALL AREA(XB(1),DAB)
      DDE=-UUG(NN)*AAG(NN)*DAE
      DDB=-UB(1)*AB(1)*DAB
      PPG(NN)=PB+(DDE+DDB)*DTP/2.+(AAG(NN)+AB(1))*(SSG(NN)-SB(1))/2.
      IF(JCH-1)35,36,36
   35 UUG(NN)=PPG(NN)-2.*AAG(NN)/(G-1.)
      GO TO 37
   36 UUG(NN)=(G-1.)*(PPG(NN)/(G+1.))
   37 CONTINUE
      IF(ABS(UUG(NN)-UGUESS)-TOL)16,16,10
   16 IF(JCH-1)23,19,19
   23 XGB(NN)=XB(1)
      XGQ(NN)=0.
      IF(UUG(NN)-AAG(NN))24,17,17
   17 JCH=1
      UUG(NN)=UUG(NN-1)
      UGUESS=UUG(NN)
      GO TO 20
   19 XGB(NN)=XB(1)
      XGQ(NN)=1.


   24 QQG(NN)=2.*AAG(NN)/(G-1.)-UUG(NN)

      RETURN
      END
```

```
      SUBROUTINE GEOM(ME,XE,DE,EE,C)
      DIMENSION XE(25),DE(25),EE(25),C(25,3)
      M1=ME-1
      DO 31 I=2,M1
      J=I-1
      C(I,2)=(DE(I)-DE(J))/(XE(I)-XE(J))
      C(I,1)=DE(I)-EE(I)*C(I,2)
   31 C(I,3)=((DE(I+1)-DE(I))/(XE(I+1)-XE(I))-C(I,2))/(4.*EE(I))
      RETURN
      END
```

```
      SUBROUTINE AREA(X,DA)
      COMMON G,DT,TOL,TOLO
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XT ,XTR,XD
      IF(X-XTL)26,26,27
   26 DA=0.
      D=DE(1)
      DD=0.
      GO TO 47
   27 CONTINUE
      IF(X-1.)29,28,28
   28 DA=0.
      D=DE(M)
      DD=0.
      GO TO 47
   29 M1=M+1
      DO 30 I=2,M1
      IF(X-XE(I))33,33,30
   30 CONTINUE
   32 DD=(DE(I-1)-DE(I))/(XE(I-1)-XE(I))
      D=DE(I-1)+DD*(X-XE(I-1))
      GO TO 46
   33 XA=XE(I-1)+EPS(I-1)
      XB=XE(I)-EPS(I)
      IF(XA-X)41,32,42
   41 IF(XB-X)44,32,32
   42 K=I-1
      GO TO 45
   44 K=I
   45 SI=X-(XE(K)-EPS(K))
      D=YC(K,1)+YC(K,2)*SI+YC(K,3)*SI**2
      DD=YC(K,2)+2.*YC(K,3)*SI
   46 DA=2.*DD/D
   47 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CROSS(IQ,KQ,JQ,JJQ,NEWQ)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      NEWQ=0
      IF(JJQ-2)12,10,10
   10 K1=NNQ(JJQ-1)
      IF(ABS(XXG(K1)-XXG(IQ))-.004)11,11,12
   11 IQ=IQ-1
      GO TO 4
   12 CONTINUE
      DO 1 I=1,NQT
      K2=NQ(I)
      IF(ABS(XG(KQ)-XG(K2))-.004)2,2,1
 1    CONTINUE
      GO TO 3
   2  LQ=IQ-1
      XXG(LQ)=XXG(IQ)
      PPG(LQ)=PPG(IQ)
      QQG(LQ)=QQG(IQ)
      AAG(LQ)=AAG(IQ)
      UUG(LQ)=UUG(IQ)
      SSG(LQ)=SSG(IQ)
      XGB(LQ)=XGB(IQ)
      XGQ(LQ)=XGQ(IQ)
      IQ=LQ
      GO TO 4
 3    DQ=(QQG(IQ)-QQG(IQ-1))/AAG(IQ-1)
      QQM(JJQ)=1.1
      CALL SHOCK(DQ,QQM(JJQ),URAT,ARAT,PRAT,DS)
      QQW(JJQ)=UUG(IQ-1)-AAG(IQ-1)*QQM(JJQ)
      XXG(IQ)=XXG(IQ-1)
      NNQ(JJQ)=IQ-1
      NEWQ=1
      XSHOCK(JJQ)=XXG(IQ)
      JJQ=JJQ+1
 4    CONTINUE
      RETURN
      END
```

```
      SUBROUTINE GCROSSQ(T,IQ,KQ,JQ,JJQ,JCS,JJCS,IIQ,KKQ,N)
      COMMON G,DT,TUL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),LCONTACT(10)
      K1=JQ-2
      K2=JQ-1
      K3=NQ(K1)
      K4=K3+1
      K5=NQ(K2)
      K6=K5+1
      AM1=(QQW(JJQ-1)+QW(JQ-1))/2.
      AM2=(QQW(JJQ-2)+QW(JQ-2))/2.
      TIN=(XSHOCK(JJQ-1)-XSHOCK(JJQ-2))/(AM2-AM1)+T
      XIN=XSHOCK(JJQ-1)+AM1*(TIN-T)
      SS4=SG(K6)
      QQ4=QG(K6)
      UU5=UG(K6)
    6 UGUESS=UU5
      DU=(UU5-UG(K3))/AG(K3)
      QM5=(-DU*(G+1.)/2.+SQRT((DU*(G+1.)/2.)**2+4.))/2.
      CALL SHOCKQ(QM5,DU5,ARAT,PRAT,DS,DQ)
      AA5=ARAT*AG(K3)
      SS5=SG(K3)+DS
      AA4=AA5*EXP((G-1.)*(SS4-SS5)/2.)
      UU4=2.*AA4/(G-1.)-QQ4
      UU5=UU4
      IF(ABS(UU5-UGUESS)-TCL)7,7,6
    7 QW5=UG(K3)-AG(K3)*QM5
      XS=XIN+QW5*(T-TIN)
      XCS=XIN+UU5*(T-TIN)
      DO 8 I=1,IQ
      IF(XXG(I)-XSHOCK(JJQ-1))8,9,9
    8 CONTINUE
      IQ=IQ+1
      GO TO 10
    9 IQ=I
   10 XXG(IQ)=XS
      PPG(IQ)=PG(K3)
      QQG(IQ)=QG(K3)
      AAG(IQ)=AG(K3)
      UUG(IQ)=UG(K3)
      SSG(IQ)=SG(K3)
      XGB(IQ)=0.
      XGQ(IQ)=0.
      JJQ=JJQ-2
      XSHOCK(JJQ)=XS
      NNQ(JJQ)=IQ
      QQW(JJQ)=QW5
      QQM(JJQ)=QM5
      JJQ=JJQ+1
      IQ=IQ+1
      XXG(IQ)=XS
```

```
      AAG(IQ)=AA5


      UUG(IQ)=UU5
      PPG(IQ)=2.*AAG(IQ)/(G-1.)+UUG(IQ)
      QQG(IQ)=2.*AAG(IQ)/(G-1.)-UUG(IQ)
      SSG(IQ)=SS5
      XGR(IQ)=0.
      XGQ(IQ)=0.
      IQ=IQ+1
      MMCS(JJCS)=IQ
      XXG(IQ)=XCS
      PPG(IQ)=PPG(IQ-1)
      QQG(IQ)=QQG(IQ-1)
      AAG(IQ)=AAG(IQ-1)
      UUG(IQ)=UUG(IQ-1)
      SSG(IQ)=SSG(IQ-1)
      XGR(IQ)=0.
      XGQ(IQ)=0.
      XCONTACT(JJCS)=XCS
      UCONTACT(JJCS)=UU4
      JJCS=JJCS+1
      IQ=IQ+1
      XXG(IQ)=XCS
      AAG(IQ)=AA4
      UUG(IQ)=UU4
      PPG(IQ)=2.*AAG(IQ)/(G-1.)+UUG(IQ)
      QQG(IQ)=2.*AAG(IQ)/(G-1.)-UUG(IQ)
      SSG(IQ)=SS4
      XGR(IQ)=0.
      XGQ(IQ)=0.
      IIQ=IQ
      KKQ=KQ
      RETURN
      END
```

```
      SUBROUTINE MOVE(NN,N,IPGO)
      COMMON G,DT,TOL,TOLQ
      COMMON XF(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      DO 14 I=1,NN
      XG(I)=XXG(I)
      PG(I)=PPG(I)
      QG(I)=QQG(I)
      AG(I)=AAG(I)
      UG(I)=UUG(I)
   14 SG(I)=SSG(I)
      N=NN
      NQT=NNQT
      DO 15 I=1,NQT
      NQ(I)=NNQ(I)
      QW(I)=QQW(I)
   15 QM(I)=QQM(I)
      NQT1=NQT+1
      DO 16 I=NQT1,10
   16 NQ(I)=0
      MCST=MMCST
      DO 17 I=1,MCST
   17 MCS(I)=MMCS(I)
      M1=MCST+1
      DO 20 I=M1,10
   20 MCS(I)=0
      IF(IPGO)19,18,19
   18 PM1=PPM1
      PW1=PPW1
   19 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CSCROSSQ(T,IQ,KQ,JQ,JJQ,JCS,JJCS,IIQ,KKQ,N)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),UCONTACT(10)
      K1=NQ(JQ-1)
      K2=MCS(JCS-1)+1
      K3=MCS(JCS-1)
      K4=K3+1
      K5=NQ(JQ-1)
      K6=K5+1
      DXDTCS=(UCONTACT(JJCS-1)+UG(K2))/2.
      DXDTS=QW(JQ-1)
      TIN=T-DT+(XG(K1)-XG(K2))/(DXDTCS-DXDTS)
      XIN=XG(K1)+DXDTS*(TIN-T+DT)
      IF(TIN-T)9,8,8
    8 TIN=T
    9 SS4=SG(K6)
      QQ4=QG(K6)
      UU5=UG(K6)
   10 UGUESS=UU5
      DU=(UU5-UG(K3))/AG(K3)
      QM5=(-DU*(G+1.)/2.+SQRT((DU*(G+1.)/2.)**2+4.))/2.
      CALL SHOCKQ(QM5,DU5,ARAT,PRAT,DS,DQ)
      AA5=ARAT*AG(K3)
      SS5=SG(K3)+DS
      AA4=AA5*EXP((G-1.)*(SS4-SS5)/2.)
      UU4=2.*AA4/(G-1.)-QQ4
      UU5=UU4
      IF(ABS(UU5-UGUESS)-TOL)11,11,10
   11 QW5=UG(K3)-AG(K3)*QM5
      PRPLOLD=2.*G*QM(JQ-1)**2/(G+1.)-(G-1.)/(G+1.)
      IF(PRAT-PRPLOLD)2,2,1
    1 WRITE(61,100)
  100 FORMAT(3X,33HCONTACT SURFACE CROSSES A Q SHOCK/)
      QMNEW=SQRT((G+1.)*((AA4/AG(K6))**(2.*G/(G-1.))+(G-1.)/(G+1.))/(2.*
     1G))
      WRITE(61,101)QMNEW
  101 FORMAT(3X,31HTHE REFLECTED WAVE IS A P SHOCK,2X,5HMACH=,E15.8)
      WRITE(61,102)PRAT
  102 FORMAT(3X,33HPRESS RATIO OF TRANSMITTED SHOCK=,E15.8)
      WRITE(61,103)PRPLOLD
  103 FORMAT(3X,30HPRESS RATIO OF INCIDENT SHOCK=,E15,8)
      WRITE(61,104)
  104 FORMAT(3X,56HTHE PROGRAM TREATS THE P SHOCK AS VERY WEAK AND DROPS
     1 IT)
      WRITE(61,105)TIN
  105 FORMAT(3X,69HIF THE P SHOCK IS NOT WEAK, ALL RESULTS ARE NOT CORRE
     1CT LATER THAN T=,E15.8)
      WRITE(61,110)
  110 FORMAT(1H1)
    2 CONTINUE
      XS=XIN+QW5*(T-TIN)
```

```
      XCS=XIN+UU5*(T-TIN)
      DO 12 I=1,IQ
      IF(XXG(I)-XSHOCK(JJQ-1))12,13,13
   12 CONTINUE
      IQ=IQ+1
      GO TO 14
   13 IQ=I
   14 XXG(IQ)=XS
      PPG(IQ)=PG(K3)
      QQG(IQ)=QG(K3)
      AAG(IQ)=AG(K3)
      UUG(IQ)=UG(K3)
      SSG(IQ)=SG(K3)
      XGB(IQ)=0.
      XGQ(IQ)=0.
      JJQ=JJQ-1
      XSHOCK(JJQ)=XS
      NNQ(JJQ)=IQ
      QQW(JJQ)=QW5
      QQM(JJQ)=QM5
      JJQ=JJQ+1
      IQ=IQ+1
      XXG(IQ)=XS
      AAG(IQ)=AA5
      UUG(IQ)=UU5
      PPG(IQ)=2.*AAG(IQ)/(G-1.)+UUG(IQ)
      QQG(IQ)=2.*AAG(IQ)/(G-1.)-UUG(IQ)
      SSG(IQ)=SS5
      XGB(IQ)=0.
      XGQ(IQ)=0.
      IQ=IQ+1
      JJCS=JJCS-1
      MMCS(JJCS)=IQ
      XXG(IQ)=XCS
      PPG(IQ)=PPG(IQ-1)
      QQG(IQ)=QQG(IQ-1)
      AAG(IQ)=AAG(IQ-1)
      UUG(IQ)=UUG(IQ-1)
      SSG(IQ)=SSG(IQ-1)
      XGB(IQ)=0.
      XGQ(IQ)=0.
      XCONTACT(JJCS)=XCS
      UCONTACT(JJCS)=UU4
      JJCS=JJCS+1
      IQ=IQ+1
      XXG(IQ)=XCS
      AAG(IQ)=AA4
      UUG(IQ)=UU4
      PPG(IQ)=2.*AAG(IQ)/(G-1.)+UUG(IQ)
      QQG(IQ)=2.*AAG(IQ)/(G-1.)-UUG(IQ)
      SSG(IQ)=SS4
      XGB(IQ)=0.
      XGQ(IQ)=0.
      IIQ=IQ
      KKQ=KQ
      RETURN
      END
```

3200 FORTRAN (2.1.0)/(RTS)

```
      SUBROUTINE SHOCKQ(QM,DU,ARAT,PRAT,DS,DQ)
      COMMON G
      DU=2.*(1.-QM**2)/((G+1.)*QM)
      ARAT=SQRT((2.+(G-1.)*QM**2)*(2.*G*QM**2-(G-1.)))/((G+1.)*QM)
      PRAT=2.*G*QM**2/(G+1.)-(G=1.)/(G+1.)
      AG1=2.*G*QM**2/(G+1.)-(G-1.)/(G+1.)
      AG2=((1.+(G-1.)*QM**2/2.)/((G+1.)*QM**2/2.))**G
      DS=ALOG(AG1*AG2)/(G*(G-1.))
      DQ=2.*(ARAT-1.)/(G-1.)+DU
      RETURN
      END
```

3200 FORTRAN (2.1.0)/(RTS)

```
      SUBROUTINE GDIS(JQ,XDISL,XDISR)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      IF(NQT)1,1,2
    1 XDISL=0.
      XDISR=1.1
      GO TO 9
    2 IF(NQT-JQ)5,3,6
    3 IF(JQ-1)4,4,8
    4 K1=NQ(JQ)
      XDISL=0.
      XDISR=XG(K1)
      GO TO 9
    5 K2=NQ(JQ-1)
      XDISL=XG(K2)
      XDISR=1.1
      GO TO 9
    6 IF(JQ-1)7,7,8
    7 K1=NQ(JQ)
      XDISL=0.
      XDISR=XG(K1)
      GO TO 9
    8 K1=NQ(JQ)
      K2=NQ(JQ-1)
      XDISL=XG(K2)
      XDISR=XG(K1)
    9 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE XDISCON(JQ,JCS,XDISR,NQOCS)
      COMMON G,DT,TOL.TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      IF(MCST)3,3,1
 1    IF(MCST-JCS)3,2,2
 2    K1=MCS(JCS)
      XRCS=XG(K1)
      GO TO 4
 3    XRCS=1.1
 4    IF(NQT-JQ)6,6,5
 5    K1=NQ(JQ+1)
      XRQS=XG(K1)
      GO TO 7
 6    XRQS=1.1
 7    IF(XRCS-XRQS)10,9,8
 8    XDISR=XRQS
      NQOCS=1
      GO TO 11
 9    XDISR=XRCS
      NQOCS=0
      GO TO 11
 10   XDISR=XRCS
      NQOCS=2
 11   CONTINUE
      RETURN
      END
```



```
      SUBROUTINE SHOCK(DEL,SM,UR,AR,PR,DS)
      COMMON G,DT,TOL
      S1=-2.
      S2=(G+1.)*(DEL+2./(G-1.))
      S3=2.
      S4=8.*G/(G-1.)
      S5=16.*G/(G-1.)**2-4.
      S6=-8./(G-1.)
      T1=S1**2-S4
      T2=2.*S1*S2
      T3=2.*S1*S3+S2**2-S5
      T4=2.*S2*S3
      T5=S3**2-S6
    1 SMG=SM
      F=T1*SM**4+T2*SM**3+T3*SM**2+T4*SM+T5
      FP=4.*T1*SM**3+3.*T2*SM**2+2.*T3*SM+T4
      SM=SMG-F/FP
      IF(ABS(SM-SMG)-TOL)2,2,1
    2 CONTINUE
      UR=ABS(2.*(1.-SM**2)/((G+1.)*SM))
      AR=SQRT((2.+(G-1.)*SM**2)*(2.*G*SM**2-(G-1.)))/((G+1.)*SM)
      PR=2.*G*SM**2/(G+1.)-(G-1.)/(G+1.)
      AG1=2.*G*SM**2/(G+1.)-(G-1.)/(G+1.)
      AG2=((1.+(G-1.)*SM**2/2.)/((G+1.)*SM**2/2.))**G
      DS=ALOG(AG1*AG2)/(G*(G-1.))
      RETURN
      END
```

3200 FORTRAN (2.1.0)/(RTS)

```
      SUBROUTINE PHUCK(IQ,N,NN)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      DIMENSION C1(1),XB(1),UB(1),AB(1),SB(1)
      PW1=PM1
      SP=PM1
   10 SGG=SP
      UA=U1+2.*A1*(SP**2-1.)/((G+1.)*SP)
      AA=SQRT((2.+(G-1.)*SP**2)*(2.*G*SP**2-(G-1.)))/((G+1.)*SP)
      PPW1=U1+A1*SP
      AG1=2.*G*SP**2/(G+1.)-(G-1.)/(G+1.)
      AG2=((1.+(G-1.)*SP**2/2.)/((G+1.)*SP**2/2.))**G
      SA=S1+ALOG(AG1*AG2)/(G*(G-1.))
      XA=XG(N)+DT*(PW1+PPW1)/2.
      CALL AREA(XA,DAA)
      C1(1)=XA-DT*(UA+AA)/2.
      CALL INTER(2,N,1,Z1,XG,C1,XB)
      CALL AREA(XB(1),DAB)
      CALL INTER(2,N,1,XG,UG,XB,UB)
      CALL INTER(2,N,1,XG,AG,XB,AB)
      CALL INTER(2,N,1,XG,SG,XB,SB)
      PB=2.*AB(1)/(G-1.)+UB(1)
      PA=PB+(-UA*AA*DAA-UB(1)*AB(1)*DAB)*DT/2.+(AA+AB(1))*(SA-SB(1))/2.
      DEL=(PA-P1)/A1
      CALL SHOCK(DEL,SP,URAT,APAT,PR,DS)
      IF(ABS(SP-SGG)-TOL)11,11,10
   11 CONTINUE
      NN=IQ+1
      XXG(NN)=XA
      PPG(NN)=PA
      UUG(NN)=UA
      AAG(NN)=AA
      QQG(NN)=2.*AA/(G-1.)-UA
      SSG(NN)=SA
      XGB(NN)=XB(1)
      XGQ(NN)=0.
      PPM1=SP
      RETURN
      END
```

3200 FORTRAN (2.1.0)/(RTS)

```
      SUBROUTINE INTER(N2,N,N1,X,Y,XS,YS)
      DIMENSION X(300),Y(300),XS(1),YS(1)
      IF(XS(1)-X(1))1,1,2
    1 J=2
      GO TO 7
    2 DO 3 I=1,N
      IF(XS(1)-X(I))4,4,3
    3 CONTINUE
      J=N
      GO TO 7
    4 J=I
      IF(ABS(X(J)-X(J-1))-.000001)6,6,7
    6 YS(1)=(YS(J)+YS(J-1))/2.
      GO TO 8
    7 YS(1)=Y(J-1)+(XS(1)-X(J-1))*(Y(J)-Y(J-1))/(X(J)-X(J-1))
    8 CONTINUE
      RETURN
      END
```

3200 FORTRAN DIAGNOSTIC RESULTS - FOR INTER

```
      SUBROUTINE PRINTOUT(T,NN,IPGO,XTEST)
      DIMENSION XT(1),UT(1),AT(1),ST(1)
      COMMON G,DT,TOL,TOLQ
      COMMON XE(25),DE(25),EPS(25),YC(25,3),M,XTL,XTR,XD
      COMMON XXG(350),PPG(350),QQG(350),AAG(350),UUG(350),SSG(350)
      COMMON XG(350),PG(350),QG(350),AG(350),UG(350),SG(350)
      COMMON Z1(350),Z2(350),XGB(350),XGQ(350)
      COMMON PM1,PW1,PPM1,PPW1
      COMMON P2P1,A1,U1,P1,Q1,S1,A2,U2,P2,Q2,S2,A3,U3,P3,Q3,S3
      COMMON NNQ(10),QQW(10),QQM(10),NQ(10),QW(10),QM(10),NNQT,NQT
      COMMON MMCS(10),MMCST,MCS(10),MCST
      COMMON XSHOCK(10)
      COMMON XCONTACT(10),LCONTACT(10)
      OP=G/(G-1.)
      XT(1)=XTEST
      WRITE(61,115)T
  115 FORMAT(//3X,5HTIME=,E15.8,1X,3H(*)//)
      WRITE(61,117)
  117 FORMAT(5X,1HI,7X,1HX,14X,1HP,14X,1HQ,14X,1HA,14X,1HU,14X,1HS,10X,8
     1HP ORIGIN,8X,8HQ ORIGIN/)
      JJQ=1
      JJCS=1
      DO 8 I=1,NN
      WRITE(61,103)I,XXG(I),PPG(I),QQG(I),AAG(I),UUG(I),SSG(I),XGB(I),
     1XGQ(I)
  103 FORMAT(3X,I3,8E15.8)
      IF(NNQT)5,5,2
    2 IF(JJQ-NNQT)3,3,5
    3 IF(I-NNQ(JJQ))5,4,5
    4 WRITE(61,102)QQM(JJQ),QQW(JJQ)
 102  FORMAT(3X,17HQ SHOCK MACH NO.=,E15.8,4X,17HQ SHOCK VELOCITY=,E15.8
     1)
      JJQ=JJQ+1
    5 IF(MMCST)6,8,6
    6 IF(JJCS-MMCST)15,15,8
   15 IF(I-MMCS(JJCS))8,7,8
    7 WRITE(61,87)
   87 FORMAT(3X,15HCONTACT SURFACE)
      JJCS=JJCS+1
    8 CONTINUE
      IF(IPGO)10,9,10
    9 WRITE(61,135)PPM1,PPW1
  135 FORMAT(/3X,20HP SHOCK MACH NUMBER=,E15.8,/3X,17HP SHOCK VELOCITY=,
     1E15.8)
   10 CONTINUE
      IF(XXG(1)-XT(1))11,12,12
   11 CONTINUE
      CALL INTER(2,NN,1,XXG,UUG,XT,UT)
      CALL INTER(2,NN,1,XXG,AAG,XT,AT)
      CALL INTER(2,NN,1,XXG,SSG,XT,ST)
      AMACH=UT(1)/AT(1)
      PRES=AT(1)**(2.*G/(G-1.))*EXP(-G*ST(1))
      WRITE(61,100)XT(1),AMACH,PRES
  100 FORMAT(//3X,6HXTEST=,E15.8,2X,5HMACH=,E15.8,2X,6HPRESS=,E15.8)
   12 WRITE(61,110)
  110 FORMAT(1H1)
      RETURN
      END
```

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APPLICATION OF THE CHARACTERISTIC METHOD IN CALCULATING THE TIME DEPENDENT, ONE-DIMENSIONAL, COMPRESSIBLE FLOW IN A TUBE WIND TUNNEL

by John D. Warmbrod and Heinz G. Struck

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